Falsifying Models of Firm Conduct

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Abstract

Econometric testing of models of firm conduct when true markups are unobserved is based on a falsifiable restriction (Berry and Haile, 2014). We reinterpret this restriction to shed light on the economic determinants of model falsifiability. We show that whether a model of conduct can be falsified largely depends on an interplay between the variation induced by the instruments, and the differences in pass-through matrices between that model and the truth. Through a set of examples that include the leading models used in empirical work, we illustrate why falsification succeeds or fails.

1 Introduction

Learning the nature of firm conduct is a fundamental goal in industrial organization (IO), as firm conduct is either the object of interest for a researcher or part of a model used to evaluate policy. Examples include investigations into the nature of vertical relationships (e.g., Villas-Boas, 2007), whether firms compete in prices or quantities (e.g., Feenstra and Levinsohn, 1995), collusion versus competitive pricing (e.g., Miller and Weinberg, 2017), intra-firm internalization (e.g., Michel and Weiergraeber, 2018), common ownership (e.g., Backus, Conlon, and Sinkinson, 2021), nonprofit conduct (e.g., Duarte, Magnolfi, and Roncoroni, 2021), and labor monopsony power (e.g., Roussille and Scuderi, 2021).

An existing econometric toolbox allows researchers to test models of conduct with market-level data when true markups are unobserved. Seminal work by Bresnahan (1982) shows that distinguishing models of conduct requires exogenous variation in market conditions, used to form instruments. In a differentiated products environment, Berry and Haile (2014)

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formalize this intuition into a falsifiable restriction and broaden the set of potential instruments. Recent work (Backus, Conlon, and Sinkinson, 2021; Duarte, Magnolfi, Sølvsten, and Sullivan, 2022) offers inferential advances in conduct testing. Duarte et al. (2022) show that reliable inference on conduct can be obtained using the test developed in Rivers and Vuong (2002) (RV) so long as the instruments are strong. Both the hypotheses of the RV test and the notion of instrument strength depend crucially on the falsifiable restriction.[1]

This falsifiable restriction is primarily a statistical condition; in this paper, we aim to better understand it from an economic point of view. We consider a differentiated products setting with unobservable demand and cost shocks, where the researcher already knows the demand system. We convert the falsifiable restriction in Berry and Haile (2014) into a condition on marginal effects of instruments on markups. This highlights two distinct effects instruments may have on markups – a direct effect when they enter the markup function, and an indirect effect through equilibrium prices.

For many commonly-used demand environments (such as logit and nested logit) and many instrument choices (such as product characteristics and rival cost shifters), the direct effect is easy to account for. In these settings, and assuming constant marginal costs, we show that a model will be falsifiable as long as the difference in inverse pass-through matrices between this model and the true model is not orthogonal to the variation in outcomes induced by the instruments. This reduces to a simple theoretical condition on the two models' pass-through matrices. This condition is more stringent for cost side instruments: within this class, any model falsified by cost side instruments is also falsified by demand side instruments. As one example, if pass-through matrices for both the true model and the model to be falsified are diagonal matrices but are different from one another, the model will be falsified by demand side instruments but not by cost side instruments.

While the importance of conduct in determining pass-through (and other economic outcomes) is well understood (Weyl and Fabinger, 2013), in this paper we establish the converse link: differences in pass-throughs is what often permits falsification of models of conduct.[2] We highlight this point by considering several examples of standard models of conduct in IO. While some knife-edge exceptions exist, we show that for most models, falsification is possible with at least one standard set of instruments.

We go on to consider falsification in settings where marginal costs may vary with the quantity produced. Falsification is typically harder in this case, as the unknown cost function can hide differences in markups. While differences in pass-throughs are still important,

[1] In Appendix A, we discuss the connection to the inferential setup in Duarte et al. (2022).

[2] In a similar spirit, Miller, Osborne, and Sheu (2017) suggest that estimates of pass-through can be used to recover a conduct parameter in a symmetric oligopoly model.

having multiple economically distinct sources of variation may be necessary for falsification. This is related to the argument in Bresnahan (1982) and Lau (1982), who emphasize the need for demand rotators in a setting without product differentiation. We show that in our setting, multiple instruments must have differential effects on either equilibrium market shares or on the difference in markups between the true model and the model to be falsified. Certain particular combinations of instruments violate this condition and therefore do not permit falsification. However, we give intuition to support the view that differentiated-products settings typically have rich enough variation to falsify most models that could be falsified if marginal costs were constant.

Our results provide a framework – grounded in economics – for applied researchers empirically testing models of conduct. Although we do not consider testing conduct in finite sample in this paper, the falsification perspective that we adopt underpins the econometric procedures that are used in empirical work. In particular, our framework can be useful for choosing instruments – enabling researchers to determine ex ante what standard sources of variation in the data could distinguish between particular candidate models, or motivating and guiding the use of new sources.

Our work relates to the nonparametric tests of the Cournot model in homogenous-goods settings proposed by Carvajal, Deb, Fenske, and Quah (2013) and Cherchye, Demuynck, and De Rock (2013). The former takes a revealed-preference approach using demand shifters but assumes no unobserved cost shocks. Closer to our approach, the latter relies on first-order conditions, checking consistency of the effect of multiple cost and demand shifters. Both find Cournot oligopoly has testable implications under mild assumptions.

Our work also relates to early literature in empirical IO which leveraged the implications of pass-through rates for testing conduct (Sumner, 1981). As noted by Bulow and Pfleiderer (1983), however, direct implications of pass-throughs for conduct depend on strong assumptions on demand, even in models of homogeneous goods. Panzar and Rosse (1987) and Ashenfelter and Sullivan (1987) derive pass-through implications of monopoly and oligopoly models that do not rely on demand estimates, and use these implications for testing. The environments considered in this early literature feature homogeneous products, and no unobservable components of demand and cost. Like Berry and Haile (2014), we consider a more general environment where testing requires estimates of demand. The link we establish between pass-through and falsifiability of models of conduct formalizes the original intuition that pass-through summarizes fundamental implications of models of firm conduct.

2 Illustrative Example

A researcher observes prices p_{jt} and market shares s_{jt} for two single-product, constant-marginal-cost firms $j = 1, 2$ selling differentiated products in multiple markets indexed by t. She already knows the demand system, and wants to know whether she can falsify a model of perfect competition, i.e., rule out both firms pricing at marginal cost in each market.

Suppose the true model generating the data is the familiar Bertrand-Nash model of competition in prices. Each firm sets price p_{jt} in market t to solve

$$\max_{p_{jt}} (p_{jt} - c_{0jt}) s_{jt}(p_t)$$

where c_{0jt} is the marginal cost faced by the firm and p_t is a vector of both firms' prices. The two firms' first-order conditions $s_{jt} + (p_{jt} - c_{0jt})\frac{\partial s_{jt}}{\partial p_{jt}} = 0$ can be rewritten in terms of the markup function, which for the true Bertrand model $m = B$ is

$$\Delta_{Bt} \quad = \quad \begin{bmatrix} p_{1t} - c_{1t} \\ p_{2t} - c_{2t} \end{bmatrix} \quad = \quad \begin{bmatrix} \frac{s_{1t}}{-\partial s_{1t}/\partial p_{1t}} \\ \frac{s_{2t}}{-\partial s_{2t}/\partial p_{2t}} \end{bmatrix}.$$

The alternative model the researcher wants to falsify is perfect competition or $m = PC$, characterized by zero markups $\Delta_{PCt} = 0$.

In an ideal environment where the researcher could directly observe actual markups Δ_{0t} (or firms' actual marginal costs c_{0t}), the markups she sees, generated by Bertrand-Nash competition, would be strictly positive, which would allow her to falsify the model of perfect competition. However, in most empirical applications the researcher does not observe either costs or markups directly. To distinguish conduct, she must instead rely on exogenous variation in cost or demand across markets.

To see how this works, suppose the researcher observed outcomes in two markets t and t' with the same demand system, and suppose firm 2's marginal cost was known to be higher in market t', $c_{02t'} > c_{02t}$, while firm 1's marginal cost was known to be the same in the two markets. She would observe different outcomes in the two markets, $p_{t'} \neq p_t$ and $s_{t'} \neq s_t$; while she knows that the difference in prices and shares is due entirely to the exogenous change in the marginal cost of firm 2, the true costs in both markets are still unobserved. Firm 1's true cost is the same across markets, or $\mathrm{d}c_{01} = c_{01t'} - c_{01t} = 0$. For any candidate model m, let c_{mt} denote the marginal costs inferred from observables under model m. The equation $\mathrm{d}c_{m1} = 0$, then, is a *falsifiable restriction*; if this restriction is violated, the model is falsified by the variation in firm 2's cost across markets.[3] Thus, in this example, a model of conduct can be falsified only insofar as the exogenous variation in rival's cost induces a change in the implied cost for a firm. If instead model m implies the same changes in marginal costs as the true model, then model m is not falsified by that exogenous source of variation.

The intuition above is in line with Bresnahan (1982) and Berry and Haile (2014).[4] While the existing literature discusses the mechanics of how to falsify a model of conduct, we seek to expand on the economic underpinnings of falsification. We will show that the economic features of a model needed for falsification are summarized in the cost pass-through implied by the model. Writing the first-order conditions for any model m in market t as

$$F_m(p_t, c_t) \quad = \quad p_t - c_t - \Delta_{mt} \quad = \quad 0 \tag{1}$$

equilibrium prices under model m are an implicit function of costs, $p_t = p_m(c_{mt})$, defined as the solution to $F_m(p_m(c_{mt}), c_{mt}) = 0$. Given a model m and observed prices p_t, the implied costs for model m, c_{mt}, are the value of c_t satisfying the first-order conditions. If we further assume that the function mapping costs to equilibrium prices, $p_m(\cdot)$, is invertible, the implied marginal costs under model m can be written as $c_{mt} = p_m^{-1}(p_t)$.

The Jacobian matrix of prices with respect to marginal costs under model m, $P_{mt} = \frac{\mathrm{d}p_m}{\mathrm{d}c_t}$, gives the effect on equilibrium prices that model m predicts would result from changes in marginal costs; we refer to P_{mt} as the *absolute cost pass-through matrix* of model m. For the true model, this implies that for a small change in marginal costs $\mathrm{d}c_{0t}$, the resulting change in prices is $\mathrm{d}p_t = P_{0t}\mathrm{d}c_{0t}$. By the inverse function theorem, the *inverse pass-through matrix* $\frac{\mathrm{d}c_m}{\mathrm{d}p_t} = P_{mt}^{-1}$ measures how the marginal costs implied by model m are affected by changes in equilibrium prices. Changes in the implied costs under model m in response to a change $\mathrm{d}c_{0t}$ in actual costs can thus be expressed as

$$\mathrm{d}c_{mt} \quad = \quad P_{mt}^{-1}\mathrm{d}p_t \quad = \quad P_{mt}^{-1}P_{0t}\mathrm{d}c_{0t}$$

(The change in true costs $\mathrm{d}c_{0t}$ induces a change $P_{0t}\mathrm{d}c_{0t}$ in observed market prices, which then induces a change $P_{mt}^{-1}P_{0t}\mathrm{d}c_{0t}$ in the marginal costs inferred under model m.)

Because the falsifiable restriction is linked to pass-throughs, we next derive these matrices for specific models. For any model m, p_m is defined implicitly by the firms' first-order

[3] Notice that the variation must be excluded from the firm's own cost. In this example, there is no useful restriction implied on firm 2's conduct.

[4] In particular, this example is a instance of Case 1 in Section 6 of Berry and Haile (2014).

conditions as shown above. The Implicit Function Theorem gives

$$P_{mt} \quad = \quad \frac{\mathrm{d}p_m(\cdot)}{\mathrm{d}c_t} \quad = \quad -\left[\frac{\mathrm{d}F_m}{\mathrm{d}p_t}\right]^{-1}\frac{\mathrm{d}F_m}{\mathrm{d}c_t} \quad = \quad (I - H_{\Delta_{mt}})^{-1} \tag{2}$$

where $H_{\Delta_{mt}} = \frac{\mathrm{d}\Delta_{mt}}{\mathrm{d}p_t}$. Recall our assumption that the true model is competition in prices, and suppose for simplicity that the demand system is simple logit, characterized by market shares

$$s_{jt} \quad = \quad \frac{\exp(x_{jt}\beta - \alpha p_{jt})}{1 + \exp(x_{1t}\beta - \alpha p_{1t}) + \exp(x_{2t}\beta - \alpha p_{2t})},$$

where x_{jt} are characteristics of product j in market t, and α and β are coefficients. In that case, we can calculate the true pass-through matrix to be

$$P_{Bt} \quad = \quad \frac{(1-s_{1t})^2(1-s_{2t})^2}{1-s_{1t}-s_{2t}}\begin{bmatrix} \frac{1}{1-s_{2t}} & \frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ \frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{1t}} \end{bmatrix}.$$

The model we wish to falsify, perfect competition, has $\Delta_{PCt} = 0$ and therefore $H_{\Delta_{PCt}} = 0$ and $P_{PCt} = P_{PCt}^{-1} = I$. For a small change in c_{02} across markets, then, the change in costs inferred under perfect competition would be

$$\mathrm{d}c_{PC} \quad = \quad P_{PCt}^{-1}P_{Bt}\mathrm{d}c_0 \quad \propto \quad I\begin{bmatrix} \frac{1}{1-s_{2t}} & \frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ \frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{1t}} \end{bmatrix}\begin{bmatrix} 0 \\ \mathrm{d}c_{02} \end{bmatrix}$$

where $\propto$ denotes proportionality. Notice that, whenever $\mathrm{d}c_{02} \neq 0$, the falsifiable restriction is violated as $\mathrm{d}c_{PC1} \neq 0$, allowing the researcher to falsify the model of perfect competition.

Economically, what happened should be clear. The true model, Bertrand-Nash competition, generates positive pass-through of rival's costs.[5] Under the alternative model, prices are equal to marginal costs, so the increase in p_1 is interpreted as an increase in c_1, violating the falsifiable restriction that c_{1t} did not change.

In this example, the two models made very different predictions – zero versus positive rival cost pass-through – making falsifiability obvious. However, the two models' predictions need not be so different for falsifiability to hold. The expression for $\mathrm{d}c_{PC}$ above suggests that whenever an off-diagonal term $[P_{mt}^{-1}P_{0t}]_{ij}$ is non-zero, a shock to product j's marginal cost will be interpreted under the wrong model as a change in product i's marginal cost. In the next section, we will establish that this is in essence what is needed for falsifiability

[5]This is not special to logit demand; under a wide range of demand systems, different firms' prices are strategic complements, so an increase in c_{2t} leads to increases in both firms' prices.

under rival cost instruments in a more general setting.

For an example where falsification is not so easily achieved, suppose that instead of competition in prices, the true model generating the data was Cournot competition (quantity-setting), with each firm simultaneously choosing market shares s_{jt} to maximize $s_{jt}(p_{jt}(s_t) - c_{0jt})$. The firms' first-order conditions $p_{jt} - c_{0jt} + s_{jt}\frac{\partial p_{jt}}{\partial s_{jt}} = 0$ give the markup function

$$\Delta_{Ct} = \begin{bmatrix} p_{1t} - c_{01t} \\ p_{2t} - c_{02t} \end{bmatrix} = \begin{bmatrix} -s_{1t}\frac{\partial p_{1t}}{\partial s_{1t}} \\ -s_{2t}\frac{\partial p_{2t}}{\partial s_{2t}} \end{bmatrix}$$

By the inverse function theorem, $\left[\frac{\mathrm{d}p_t}{\mathrm{d}s_t}\right] = \left[\frac{\mathrm{d}s_t}{\mathrm{d}p_t}\right]^{-1}$; in the case of simple logit demand, we can calculate the pass-through matrix for the Cournot model as

$$P_{Ct} = (I - H_{\Delta_{Ct}})^{-1} = \begin{bmatrix} \frac{1-s_{1t}-s_{2t}}{1-s_{2t}} & 0 \\ 0 & \frac{1-s_{1t}-s_{2t}}{1-s_{1t}} \end{bmatrix}$$

While the result is rather knife-edge and specific to logit demand, the Cournot model predicts zero pass-through of rival costs. This means we can't falsify perfect competition based on variation in rival costs when the true model is Cournot. A change in firm 2's true cost c_{02} across markets would induce a change

$$\mathrm{d}c_m = P_{mt}^{-1}P_{0t}\mathrm{d}c_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1-s_{1t}-s_{2t}}{1-s_{2t}} & 0 \\ 0 & \frac{1-s_{1t}-s_{2t}}{1-s_{1t}} \end{bmatrix}\begin{bmatrix} 0 \\ \mathrm{d}c_{02} \end{bmatrix}$$

in the costs implied by model m, with $\mathrm{d}c_{m1} = 0$ *satisfying* the falsifiable restriction. Mathematically, since P_{mt} and P_{0t} are both diagonal matrices, $P_{mt}^{-1}P_{0t}$ is diagonal as well. Economically, since Cournot competition implies zero pass-through of rival costs, a change in c_2 leads to no change in p_1 across markets, leading to no change in the marginal cost inferred for product 1 under perfect competition. As we'll formalize below, more generally, when $P_{mt}^{-1}P_{0t}$ is a diagonal matrix, rival cost shifters can't falsify model m when the truth is model 0.

These two examples highlight an important point: falsifiability of a model using exogenous variation is not about the *level* of markups, but the *slope* (how they respond to changes in the exogenous variables). The Cournot model predicts higher markups than the Bertrand model,[6] and in this sense is "further away" from the model of perfect competition. However,

[6]This can be shown to hold much more generally; in the two-firm case, firm 1's markups under the two models are $\Delta_{Ct1} = -s_{1t}\frac{\partial s_{2t}}{\partial p_{2t}}\left(\frac{\partial s_{1t}}{\partial p_{1t}}\frac{\partial s_{2t}}{\partial p_{2t}} - \frac{\partial s_{1t}}{\partial p_{2t}}\frac{\partial s_{2t}}{\partial p_{1t}}\right)^{-1}$ and $\Delta_{Bt1} = -s_{1t}\left(\frac{\partial s_{1t}}{\partial p_{1t}}\right)^{-1}$, and $\Delta_{Ct1} > \Delta_{Bt1}$ easily follows as long as $\frac{\partial s_{1t}}{\partial p_{2t}}$ and $\frac{\partial s_{2t}}{\partial p_{1t}}$ are nonzero and have the same sign.

because the pass-through patterns under perfect competition are more similar to those of the Cournot model than the Bertrand, perfect competition can be falsified under the latter model but not under the former.

In the case of Cournot competition, then, how should we proceed? Even when all off-diagonal terms of $P_{mt}^{-1}P_{0t}$ are zero and therefore rival cost shifters can't falsify the model, exogenous variation in product characteristics can still work in many cases. Under simple logit, product characteristics and price enter demand only through an index $\delta_{jt} = x_{jt}\beta - \alpha p_{jt}$ which shifts the mean consumer value for good j in market t. In that case, a change in a product characteristic has the same effect on markups as a change in costs – specifically, $\frac{\mathrm{d}\Delta_t}{\mathrm{d}x_{jt}^{(k)}} = -\frac{\beta^{(k)}}{\alpha}\frac{\mathrm{d}\Delta_t}{\mathrm{d}c_{jt}}$. Thus, if the Cournot model predicts no pass-through of rivals' costs, it also predicts no pass-through of *rivals'* product characteristics. However, exogenous variation in characteristics of product j itself can serve to falsify conduct insofar as they are excluded from j's marginal cost. Under the Cournot model, a product's characteristics will shift that same product's price; as long as the change in characteristics is independent of cost, this will falsify the perfect competition model. More generally, for demand systems where product characteristics and price enter demand only through mean utility,[7] we'll establish that model m can typically be falsified by variation in product characteristics as long as $P_{mt}^{-1}P_{0t}$ is not equal to the identity matrix.

Again, in the current example, falsifiability is obvious, since in the model to be falsified product characteristics excluded from cost should not move prices at all, while they do in the true model. However, once again, a model need not make such starkly "wrong" predictions to be falsifiable. As long as model m and the true model make different predictions about pass-through, the changes in equilibrium prices induced by changes in each product characteristic under the true model cannot be rationalized by model m, and product characteristics excluded from costs suffice to falsify model m.

In the next section, we formalize our results in a more general setting using exogenous variation to construct instruments. We return in Section 4 to additional examples of commonly-used models of competition for more intuition on what is behind falsifiability.

3 General Framework

3.1 Environment and Notation

We consider falsification of models of firm conduct using data across many markets. A set of multi-product firms compete in each market t; for simplicity, we assume the same set of J

[7] When characteristics or price enter outside mean utility, a more complicated result, Proposition 1, holds.

products is sold in every market, although their characteristics may differ across markets. For each product and market combination (j, t), the researcher observes price p_{jt}, market share s_{jt}, a vector of product characteristics x_{jt}, and a vector of cost shifters w_{jt} that affects the product's marginal cost.[8] For any variable a_{jt}, let a_t denote the vector of values for all products j in market t. We assume that, for all markets t, the demand system is $s_t = \mathscr{s}(p_t, x_t, \xi_t)$, where ξ_t is a vector of unobserved product characteristics. To focus on the supply side, we assume that the demand system is already known to the researcher.[9] We normalize market size to 1, so that quantity q_{jt} and market share s_{jt} can be used interchangeably.

The data in each market t are generated by equilibrium play in some true model of firm behavior, characterized by a system of first-order conditions,

$$p_t = \Delta_{0t} + c_{0t}, \tag{3}$$

where Δ_{0t} is the true vector of markups in market t and c_{0t} is the true vector of marginal costs. Under each model m, we can calculate the implied markups Δ_{mt} as a known function of observables and demand primitives. Implied marginal costs c_{mt} can therefore also be calculated via a model-specific version of the first-order conditions in Equation (3), as $p_t - \Delta_{mt}$.

For the first-order conditions of any model m to characterize a well-defined empirical model, we require the following, analogous to Assumption 13 in Berry and Haile (2014):

Assumption 1. *(Equilibrium Uniqueness)* For any model m, including the true model, either there exists a unique equilibrium, or the equilibrium selection rule is such that the same p_t arises whenever the vector (c_{mt}, x_t, ξ_t) is the same.

3.2 An Econometric Perspective on Falsifiability

Unlike the example of Section 2, researchers typically cannot hold demand and all but one cost fixed; unobserved shocks will vary across markets along with any instrument. Following the literature, we separate observable and unobservable parts of marginal costs by assuming that costs are separable in a function of observable cost shifters w_{jt} and quantities, and in an unobserved shock, or $c_{0jt} = \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) + \omega_{0jt}$. While the researcher can control for w_{jt}, she cannot keep ω_{0jt} fixed across markets. Instead, the researcher can construct instruments z_{jt} that are mean independent of the unobserved cost shocks under the true model. Formally, we assume that the instruments z_{jt} satisfy the following exclusion restriction:

[8] Although x_{jt} and w_{jt} could include the same variables, we maintain for simplicity that they are distinct.
[9] See Berry and Haile (2014) for conditions by which the demand system is nonparametrically identified.

Assumption 2. *(Instrument Exogeneity)* Marginal costs are $c_{0jt} = \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) + \omega_{0jt}$ for each j, and z_{jt} is a vector of K excluded instruments such that $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$.

Berry and Haile (2014) provide several sources of variation that can be used to construct instruments. We will focus on two of these, rival cost shifters and own and rival product characteristics, as these are standard sources of variation used in empirical work and illustrate the main forces behind falsification. We refer throughout to *cost side instruments* and *demand side instruments*, respectively, as those formed with this variation.

For now, we simplify the environment with the following assumption:

Assumption 3. *(Constant Marginal Cost)* Marginal costs are constant in quantities and only depend on the observable cost shifters w_{jt}, or $\bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) = \bar{c}_{0j}(\mathrm{w}_{jt})$ for all j.

In Section 5 we remove this assumption and extend the results to any cost structure where ω_{0jt} is additively separable.

When true markups are unobserved, what we know about the true model is that the condition $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ holds for the true cost function. For a candidate model m and candidate cost function $\bar{c}_{mj}$, we can define ω_{mjt}, a model specific analog of ω_{0jt}, as $\omega_{mjt} = p_{jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt})$; the expression $E[\omega_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ is a falsifiable restriction, the analogue of the condition in Theorem 9 of Berry and Haile (2014). We will say a model m is *falsified by the instruments* z_{jt} if there are no cost functions $\{\bar{c}_{mj}(\mathrm{w}_{jt})\}_{j=1}^{J}$ satisfying this falsifiable restriction almost surely over the values of w_{jt} and z_{jt}.

As prices in the data are generated from the true model, we can rewrite this expression as $\omega_{mjt} = \Delta_{0jt} - \Delta_{mjt} + \bar{c}_{0j}(\mathrm{w}_{jt}) - \bar{c}_{mj}(\mathrm{w}_{jt}) + \omega_{0jt}$. From this, we can restate the falsifiable restriction as follows.[10]

Lemma 1. *Under Assumptions 1-3, model m is falsified by instruments z_{jt} if and only if for some j there exists no function $\bar{c}_{mj}$ such that*

$$E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = \bar{c}_{mj}(\mathrm{w}_{jt}) - \bar{c}_{0j}(\mathrm{w}_{jt}) \qquad a.s.$$

Here, as in the rest of the section, the expectation is taken over realizations of unobservables, and *almost surely (a.s.)* refers to realizations of the exogenous observables $(\mathrm{w}_{jt}, z_{jt})$.

The restriction in Lemma 1 is a statistical condition for falsification of a model of conduct, but it does not speak to the features of a model that allow falsification. To begin to address this question, it is useful to restate Lemma 1 in terms of the marginal impacts of the instruments on markups. To do this, we will assume markups do not move discontinuously as instruments change:[11]

[10] Proofs of all lemmas, propositions, and corollaries are in Appendix B.

[11] Assumption 4 holds for all models in this paper, and for all standard models.

Assumption 4. *(Continuous Markups)* For any model m, for any j, $E[\Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]$ is absolutely continuous in z_{jt}.

Based on Lemma 1, for a model not to be falsified, the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ must match the difference in implied cost at each value of the instruments z_{jt}. As instruments are excluded from cost, a marginal change in any of the K instruments has no effect on the implied costs for either model. This means that if a model is not falsified, the impact of the instruments on the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ must be zero. We focus on the limit of this difference,

$$\lim_{h \to 0} \frac{E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt} + h_k] - E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt}]}{h}$$

where h_k is a K-vector of zeros with the scalar h in the k-th position. The marginal effect of the k-th instrument $z_{jt}^{(k)}$ on the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ is the average difference in the marginal effect of $z_{jt}^{(k)}$ on Δ_{0jt} and Δ_{mjt}, giving the following:

Lemma 2. *Under Assumptions 1-4, model m is falsified by instruments z_{jt} if and only if for some j and k,*

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

The derivative of markups with respect to the instruments is an object whose properties depend on the economics of the model, so this lemma allows us to connect the econometric perspective on falsifiability with a more theoretical view inspired by the example in Section 2. To falsify a model, variation in the instruments must induce differential changes in the implied markups for that model and the true model. However, since demand and cost shocks are unobserved, it is not possible to isolate changes solely caused by the instruments; we therefore need the marginal impact of the instruments to differ across the two models when we average over the unobserved shocks. Having restated the falsifiable restriction in terms of marginal effects, we now explore the relevant economic features for falsifiablility.

3.3 Role of Pass-Through

In the example in Section 2, falsifiability depended on pass-through of costs. We now show that pass-through also helps determine falsifiability in a more general setting. To see this, we note that the markup function for either model is a function of two endogenous variables, market shares and prices. As the demand system makes market shares a function of prices, we can write the vector Δ_{mt} as a function of prices, instruments and other exogenous variables,

$$\Delta_{mt} = \Delta_m(p_t, z_t, \mathrm{w}_t, x_t, \omega_{mt}, \xi_t).$$

Instruments may thus affect markups either directly, or through their effect on prices.

In light of Lemma 2, we are interested in the average difference in the causal effects of an instrument $z_{jt}^{(k)}$ on model m and the truth. Letting $(A)_j$ denote the j^{th} row of a vector or matrix, we have

$$\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \quad = \quad \underbrace{\frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}}}_{\text{Direct Effect}} + \underbrace{\left(\frac{\partial\Delta_{0t}}{\partial p_t} - \frac{\partial\Delta_{mt}}{\partial p_t}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}}_{\text{Indirect Effect}}$$

where $p_0(\cdot)$ is the function mapping primitives to equilibrium prices under the true model. From this expression we see two distinct effects of instruments on the difference in markups. The first is the *direct effect* of instruments, or $\frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}}$. This element is non-zero whenever instruments such as product characteristics differentially enter the markup functions of model m and the truth. The second term represents the *indirect effect*, which happens through prices. As this term is more complex, we further investigate its economic content.

For any model m we can compute the cost pass-through matrix P_{mt} as in Section 2 via the Implicit Function Theorem. Note that we can rewrite the difference in price derivatives of markups above as a function of inverse pass-through because $P_{mt}^{-1} = \left(I - \frac{\partial\Delta_{mt}}{\partial p_t}\right)$. Inverse pass-through matrices are well-defined under the following assumption:

Assumption 5. *(Invertibility of Pass-throughs)* For any model m, in any market t, the pass-through matrix P_{mt} has full rank.

This assumption has economic content, as it requires that each product has non-zero pass-through for at least one cost (either own or rival) in the market. Moreover, pass-through vectors for each product cannot be linear combinations of those of other products, meaning that costs must affect different products in a distinct way. This is satisfied in most models, as the pass-through of own cost that is measured by the main diagonal of P_{mt} is typically different than the pass-through of rival costs. We illustrate the content of this assumption in Section 4.2.

We can then establish the following:

Proposition 1. *Under Assumptions 1-5, a model m is falsified by instruments z_{jt} if and only if for some j and k,*

$$E\left[\frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}} + \left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p. \tag{4}$$

This proposition casts falsification in terms of the direct and indirect effect of the instruments. When the instruments move the markup functions directly and differentially, this will likely enable falsification. Furthermore, if instruments affect prices, falsification depends on $(P_{mt}^{-1} - P_{0t}^{-1})$, the difference in inverse pass-throughs of the true model and model m. Similar to the intuition in the two-firm example in Section 2, differences in inverse pass-throughs allow the variation in prices induced by the instruments to result in different implied costs.

Note that the difference in inverse pass-throughs is multiplied by $\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}$, the variation in prices induced by the instruments. Differences in pass-throughs only matter for falsification if instruments move prices. In turn, this depends on the pricing function of the true model of conduct p_0. Taken together, the term $\left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}$ represents the indirect effect of the instruments through prices.

To gain further insight, we interpret Proposition 1 separately for cost side instruments (based on rival cost shifters) and demand side instruments (based on product characteristics). To do that, we restrict the nature of markup functions according to the following assumption:

Assumption 6. *(No Abnormal Effects of Primitives)* For any model m, the markup function Δ_m (i) depends on costs only through their effect on equilibrium prices, and (ii) depends on product characteristics only through their effect on shares s_t and demand derivatives $\frac{\partial s_t}{\partial p_t}$.

As we show in Appendix C, this assumption holds in a variety of models where firms maximize profits under various assumptions on what their rivals are holding fixed; or if firms maximize any weighted sum of their own profits, other firms' profits, and consumer surplus (or total welfare). It also holds in different variations of the price or quantity leadership model where some firms move first and others, seeing their actions, move second.[12]

Falsifiability with cost side instruments

The implications of the proposition are particularly stark when the instruments are formed with rival cost shifters. Under Assumption 6, the instruments have no direct effect on

[12] Where it fails is with "cost-plus" pricing – markups which are a fixed percentage of costs – or when firms' objective functions are something other than profits (for example, revenue, or a weighted average of profit and revenue). We discuss falsification for these models in Section 4.2.

markups, and enter either markup function only through $p_0(\cdot)$. For simplicity, we further assume that the true marginal cost is linear in cost shifters:

Assumption 7. *(Linear Marginal Cost)* $\bar{c}_{0j}(\mathrm{w}_{jt}) = \tau \mathrm{w}_{jt}$, with all elements of τ nonzero.

Next, we define the $J \times J$ matrix $[P_m^{-1}P_0]^\star$ by

$$[P_m^{-1}P_0]_j^\star \quad = \quad E\left[P_{mt}^{-1}P_{0t}|\mathrm{w}_{jt}, z_{jt}\right]_j$$

That is, $[P_m^{-1}P_0]^\star$ is the expected value of $P_{mt}^{-1}P_{0t}$, but where the expectation in the j-th row is taken conditional on the realized values of product j's cost shifters and instruments $(\mathrm{w}_{jt}, z_{jt})$. Thus, $[P_m^{-1}P_0]^\star$ is a function of the full vector of observables (w_t, z_t), with each row depending on a different subset of those observables. This matrix arises from Equation (4) because $\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}$ will often be proportional to P_{0t}, so $(P_{mt}^{-1} - P_{0t}^{-1})\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \propto (P_{mt}^{-1}P_{0t} - I)$.

The example in the previous section suggested that falsifiability of a model m depends on the characteristics of the matrix $P_{mt}^{-1}P_{0t}$. In the richer setting with observable and unobservable variation, falsifiability depends on the characteristics of $[P_m^{-1}P_0]^\star$:

Corollary 1. *Suppose that for each product j, the vector of instruments z_{jt} includes a cost shifter of every rival product. Under Assumptions 1-7, model m is falsified by the instruments if with positive probability over (w_t, z_t), the matrix $[P_m^{-1}P_0]^\star$ is not diagonal.*

The intuition for this result is that for $\ell \neq j$, $[P_{mt}^{-1}P_{0t}]_{j,\ell}$ is proportional to the marginal effect of product ℓ's marginal cost $c_{0\ell t}$ on product j's inferred cost under model m, c_{mjt}. Thus, if for some values of observables, $[P_{mt}^{-1}P_{0t}]_{j,\ell}$ is nonzero in expectation, then for those observables a change in a shifter of $c_{0\ell t}$ would imply under model m a change in the mean of c_{mjt}, violating the restriction that ω_{mjt} is mean-independent of the instruments.[13]

It's instructive to note how the conditions of this corollary could fail. $[P_m^{-1}P_0]^\star$ could be diagonal when the variation in rival costs does not induce variation in the vector of equilibrium prices p_t, as P_{0t} would be the zero matrix, which is however ruled out by Assumption 5. It could also be diagonal if in all markets, $P_{mt} = P_{0t}$, or if P_{mt} and P_{0t} are always both diagonal matrices. In this latter case, changes in a rival's cost only affect that rival's price as P_{0t} is diagonal, meaning that the price of product j is not changed by the variation in the instrument. However, because P_{mt}^{-1} is also diagonal, only variation in the price of product j can induce variation in the implied cost of product j. Thus, the difference in the implied cost of product j under model m and the truth is zero and the model cannot be falsified by cost side instruments.

[13] Falsifiability does not require every product's instruments z_{jt} to include a shifter of every rival product's cost, just that there be some pair (j,ℓ), $\ell \neq j$, where $[P_m^{-1}P_0]_{j,\ell}^\star \neq 0$ and z_{jt} includes a shifter of $c_{0\ell}$.

Falsifiability with demand side instruments

Two key differences arise between the case of cost side instruments just discussed and the case of demand side instruments. The first is that, since the product characteristics are excluded from cost, the researcher can use variation in a product's own characteristics as a valid instrument as well as variation in rival product characteristics. Second, product characteristics in general will directly enter the markup function and therefore have a direct effect on the implied costs under model m, so that $\frac{\partial \Delta_{mj}}{\partial z_{jt}^{(k)}} \neq 0$.

Under an additional assumption on the model of demand, the effect of product characteristics becomes very similar to the effect of cost shifters on markups:

Assumption 8. *(Demand Index)* Demand depends on x_t and p_t only through δ_t, or $s_t = \mathscr{s}(x_t, p_t, \cdot) = \mathscr{s}(\delta_t, \cdot)$, where $\delta_t = x_t\beta - \alpha p_t + \xi_t$, $\alpha > 0$, and all elements of β are nonzero.

This assumption is satisfied when the demand system is logit or nested logit; it is not satisfied for mixed logit demand models with random coefficients on either price or characteristics, but we expect the added variation in richer models to make falsification easier, not harder, as we discuss in Section 4.3. Combined with Assumption 6, Assumption 8 implies that x_t and c_t affect equilibrium markups only through the term $x_t\beta - \alpha c_t$, and therefore that the effects of marginal costs and product characteristics on markups are identical.[14]

Under this assumption, we can derive a corollary for falsification with product characteristics that is very similar to the one for rival cost shifters.

Corollary 2. *Suppose that for every product j, the vector of instruments z_{jt} includes a product characteristic of every product. Under Assumptions 1-6 and 8, model m is falsified by the instruments z_t if with positive probability over (w_t, x_t), the matrix $[P_m^{-1}P_0]^\star$ is not equal to the identity matrix.*

The intuition is that the marginal effect of a product characteristic $x_{\ell t}^{(k)}$ on the inferred costs of product j under model m is proportional to $(I - P_{mt}^{-1}P_{0t})_{j\ell}$. Thus, if this term is nonzero in expectation for some values of observables, a change in $x_{\ell t}^{(k)}$ would lead to changes in the mean of c_{mjt}; since product characteristics used as instruments are excluded from costs, this would again violate the restriction that ω_{mjt} is mean-independent of the instruments.

Loosely, while falsifying a model with cost side instruments requires that $P_{mt}^{-1}P_{0t}$ is not diagonal on average, falsifying a model with demand side instruments only requires that

[14] If demand depends on x_{jt} and p_{jt} only through δ_{jt}, then under the Bertrand model, we can think of a single-product firm directly choosing markup Δ_{jt} to maximize $(p_{jt} - c_{jt})\mathscr{s}_j(\delta_{jt}, \delta_{-jt}) = \Delta_{jt}\mathscr{s}_j\left(x_{jt}\beta - \alpha c_{jt} + \xi_{jt} - \alpha\Delta_{jt},\, \delta_{-jt}\right)$, and therefore optimal Δ_{jt} depends on market primitives only through the term $x_{jt}\beta - \alpha c_{jt} + \xi_{jt}$; this easily extends to all the models discussed in Appendix C.

$P_{mt}P_{0t}^{-1} \neq I$. Thus, when α, β, and τ are all non-zero, it is easier to falsify model m using product characteristics: under Assumptions 1-8 any model which is falsified by rival cost shifters is also falsified by product characteristics.

4 Falsifiability of Various Models of Conduct

Here we explore the implications of Proposition 1 by examining a range of models of conduct considered by applied researchers in IO. We start with several examples where models can be falsified with either cost or demand side instruments. The key feature of these examples is that the degree of pass-through of rivals' costs differs across the two models. Reassuringly, this set comprises many of the most commonly considered models in the literature. However, not all models can be falsified with these instruments; we show some meaningful economic examples where falsification with one or both sources of variation breaks down, which gives helpful intuition for what is actually driving falsifiability.

For both tractability and ease of exposition, as in Section 2, we focus on the case of two single-product firms facing simple logit demand in each market t. We discuss at the end of the section how results extend to more general settings. Recall that for each model m, prices, implied markups and implied costs in market t are characterized by the first-order conditions in Equation (1), and the pass-through matrix can be obtained via Equation (2).

4.1 Models Falsifiable by either Type Of Instrument

Corollaries 1 and 2 show that a model m can be falsified against the truth by either cost side or demand side instruments if for some realizations of observables (w_t, z_t), the matrix $[P_m^{-1}P_0]^\star$ is not a diagonal matrix. Here, we show that many of the standard models considered in IO satisfy this condition.

Example: price versus quantity competition

Result 1. *If the true model is competition in prices, then competition in quantities can be falsified with either type of instrument, and vice versa.*

These two models were discussed in Section 2. In the Bertrand-Nash model of price competition, firms set prices p_{jt} to maximize profits. In the case of two single-product firms and logit demand, this leads to markups, inverse pass-through, and pass-through matrices

$$\Delta_{Bt} = \begin{bmatrix} \frac{1}{\alpha(1-s_{1t})} \\ \frac{1}{\alpha(1-s_{2t})} \end{bmatrix}, \quad P_{Bt}^{-1} = \begin{bmatrix} \frac{1}{1-s_{1t}} & -\frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ -\frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{2t}} \end{bmatrix}, \quad \text{and} \quad P_{Bt} = \kappa_{Bt}\begin{bmatrix} \frac{1}{1-s_{2t}} & \frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ \frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{1t}} \end{bmatrix}$$

where $s_{0t} = 1 - s_{1t} - s_{2t}$ and $\kappa_{Bt} = s_{0t}^{-1}(1-s_{1t})^2(1-s_{2t})^2$. In the Cournot model, on the other hand, firms choose quantities, taking the other firm's quantity as given, to maximize profits, leading to markups, inverse pass-through, and pass-through

$$\Delta_{Ct} = \begin{bmatrix} \frac{1-s_{2t}}{\alpha s_{0t}} \\ \frac{1-s_{1t}}{\alpha s_{0t}} \end{bmatrix}, \quad P_{Ct}^{-1} = \begin{bmatrix} \frac{1-s_{2t}}{s_{0t}} & 0 \\ 0 & \frac{1-s_{1t}}{s_{0t}} \end{bmatrix}, \quad \text{and} \quad P_{Ct} = \begin{bmatrix} \frac{s_{0t}}{1-s_{2t}} & 0 \\ 0 & \frac{s_{0t}}{1-s_{1t}} \end{bmatrix}$$

Thus, the off-diagonal terms of $P_{Bt}^{-1}P_{Ct}$ are always negative, so they can't be zero in expectation. Likewise, the off-diagonal terms of $P_{Ct}^{-1}P_{Bt}$ are always positive. This gives the result.

The intuition is as in the first example of Section 2. When the true model is Bertrand, an increase in firm 2's marginal cost increases both firms' equilibrium prices. Since Cournot predicts no pass-through of rival costs, the Cournot model interprets these changes as increases in both firms' costs, falsifying the restriction. When the true model is Cournot, an increase in c_{02t} leads to an increase only in firm 2's price; since the Bertrand model predicts positive rival cost pass-through, it interprets this change as an increase in firm 2's cost and an offsetting decrease in firm 1's, violating the falsifiable restriction that c_{m1t} has not changed.

Example: competition versus joint profit maximization

Result 2. *If the two firms are colluding perfectly to jointly maximize profits, the Bertrand model and Cournot model can both be falsified with either type of instrument. Similarly, if the true model is either price or quantity competition, perfect collusion can be falsified with either type of instrument.*

Consider the model where the two firms collude perfectly, jointly setting both products' prices to maximize combined profits. The problem

$$\max_{p_{1t},p_{2t}} \{(p_{1t} - c_{1t})s_{1t}(\cdot) + (p_{2t} - c_{2t})s_{2t}(\cdot)\}$$

leads to a Joint Profit Maximization markup function, inverse pass-through matrix, and pass-through matrix

$$\Delta_{Jt} = \begin{bmatrix} \frac{1}{\alpha s_{0t}} \\ \frac{1}{\alpha s_{0t}} \end{bmatrix}, \quad P_{Jt}^{-1} = \frac{1}{s_{0t}} \begin{bmatrix} 1-s_{2t} & s_{2t} \\ s_{1t} & 1-s_{1t} \end{bmatrix}, \quad \text{and} \quad P_{Jt} = \begin{bmatrix} 1-s_{1t} & -s_{2t} \\ -s_{1t} & 1-s_{2t} \end{bmatrix}$$

Above, we saw that the Bertrand pass-through matrix P_{Bt} has all positive elements, and its inverse, P_{Bt}^{-1}, has positive diagonal elements and negative off-diagonals. From this, we can easily see that $P_{Jt}^{-1}P_{Bt}$ has all positive elements, so its off-diagonals can't be zero in expectation; and that $P_{Bt}^{-1}P_{Jt}$ always has negative off-diagonal elements, which therefore again can't be zero in expectation. Similarly, since the Cournot pass-through matrix P_{Ct} is diagonal, $P_{Jt}^{-1}P_{Ct}$ will always have positive off-diagonal elements, and $P_{Ct}^{-1}P_{Jt}$ will always have negative off-diagonal elements, so neither matrix can be diagonal in expectation.

Here, it is clear why we can falsify any of these models. Bertrand competition predicts positive rival cost pass-through, Cournot predicts zero rival cost pass-through, and perfect collusion predicts *negative* rival cost pass-through, so it's hardly surprising that a shock to rival costs distinguishes any two of these models. The next example shows that a model can be falsifiable even when its pass-through prediction is much closer to that of the true model.

Example: weighted Cournot competition

IO economists often use profit weights to model collusion or common ownership (e.g., Backus et al., 2021). Consider a model where firms compete in quantities, but instead of maximizing its own profit, each firm maximizes a weighted sum of its own and the other firm's profits. Specifically, each firm j chooses quantity s_{jt} to solve

$$\max_{s_{jt}} \left\{ s_{jt}(p_{jt}(\cdot) - c_{jt}) + \theta_j s_{-jt}(p_{-jt}(\cdot) - c_{-jt}) \right\}$$

where $-j$ refers to the identity of the rival firm. This nests standard Cournot competition (when $\theta_1 = \theta_2 = 0$) and perfect collusion/joint profit maximization (when $\theta_1 = \theta_2 = 1$), along with intermediate cases that might be interpreted as "imperfect collusion."

Result 3. *If the true model is Cournot competition with profit weights, then a weighted Cournot model with misspecified profit weights can be falsified with either type of instruments.*

Simplifying the first-order conditions, we can work out the markups, inverse pass-through matrix, and pass-through matrix

$$\Delta_{WCt} = \begin{bmatrix} \frac{1-(1-\theta_1)s_{2t}}{\alpha s_{0t}} \\ \frac{1-(1-\theta_2)s_{1t}}{\alpha s_{0t}} \end{bmatrix}, \; P_{WCt}^{-1} = \frac{1}{s_{0t}} \begin{bmatrix} 1 - s_{2t} & \theta_1 s_{2t} \\ \theta_2 s_{1t} & 1 - s_{1t} \end{bmatrix}, \; P_{WCt} = \frac{s_{0t}}{\kappa_{WCt}} \begin{bmatrix} 1 - s_{1t} & -\theta_1 s_{2t} \\ -\theta_2 s_{1t} & 1 - s_{2t} \end{bmatrix}$$

where $\kappa_{WCt} = (1 - s_{1t})(1 - s_{2t}) - \theta_1\theta_2 s_{1t}s_{2t}$. (As expected, the markups coincide with Δ_{Ct} when $\theta_1 = \theta_2 = 0$, and with Δ_{Jt} when $\theta_1 = \theta_2 = 1$.)

Suppose the true model is weighted Cournot competition with weights $\theta_0 = (\theta_{01}, \theta_{02})$, and we are interested in falsifying a model of weighted Cournot with misspecified weights $\theta_m =$

$(\theta_{m1}, \theta_{m2}) \neq \theta_0$. Focusing on the off-diagonal terms and dropping constants, we can calculate

$$P_{WCmt}^{-1} P_{WC0t} \quad \propto \quad \begin{bmatrix} \star & (\theta_{m1} - \theta_{01}) s_{2t}(1 - s_{2t}) \\ (\theta_{m2} - \theta_{02}) s_{1t}(1 - s_{1t}) & \star \end{bmatrix}$$

Thus, if $\theta_{m1} > \theta_{01}$, the top-right off-diagonal is always positive, and therefore positive in expectation; if $\theta_{m1} < \theta_{01}$, it's always negative, hence negative in expectation. Likewise, if $\theta_{m2} > \theta_{02}$, the bottom-right term is always positive, and if $\theta_{m2} < \theta_{02}$ always negative. This gives the result.

If both the true model and the model to be falsified predict negative pass-through of rival's costs, how are we able to falsify the wrong model? If we let P_{mtij} denote the pass-through of firm j's costs to firm i's equilibrium price in market t under model m, so that

$$P_{mt}^{-1} P_{0t} \quad = \quad \begin{bmatrix} P_{mt11} & P_{mt12} \\ P_{mt21} & P_{mt22} \end{bmatrix}^{-1} \begin{bmatrix} P_{0t11} & P_{0t12} \\ P_{0t21} & P_{0t22} \end{bmatrix}$$

we can do some matrix algebra and calculate the off-diagonal terms of $P_{mt}^{-1} P_{0t}$ as

$$P_{mt}^{-1} P_{0t} \quad = \quad \begin{bmatrix} \star & k_{1t} \left(\frac{P_{0t12}}{P_{0t22}} - \frac{P_{mt12}}{P_{mt22}} \right) \\ k_{2t} \left(\frac{P_{0t21}}{P_{0t11}} - \frac{P_{mt21}}{P_{mt11}} \right) & \star \end{bmatrix}$$

where $k_{it} = \frac{1}{P_{mt11} P_{mt22} - P_{mt21} P_{mt12}} \frac{P_{mtii} P_{0tii}}{P_{mt11} P_{0t11} P_{mt22} P_{0t22}}$. A model being falsifiable by rival cost shifters thus depends on whether it makes a different prediction than the true model about the pass-through of firm j's marginal costs to firm i's equilibrium price, *relative to* the pass-through to firm j's own equilibrium price. As long as this ratio is different between the two models, falsification is possible with cost side instruments.[15]

Example: linear versus two-part fees for vertical supply

Next, we depart from our duopoly setting, and instead consider a monopolist in a vertical relationship. Specifically, there is a single product, produced by an upstream monopolist with marginal cost c_{Ut} and sold by a downstream monopolist with marginal cost c_{Dt} in addition to the wholesale price p_{Ut}. Demand is single-product logit – the product's market share is

$$s_{Ut} \quad = \quad s_{Dt} \quad \equiv \quad s_t \quad = \quad \frac{\exp(x_t \beta - \alpha p_{Dt} + \xi_t)}{1 + \exp(x_t \beta - \alpha p_{Dt} + \xi_t)}$$

[15] With more than 2 products, $P_{mt}^{-1} P_{0t}$ not being diagonal is the same as P_{0t} not being a diagonal matrix times P_{mt}; thus, falsifiability requires each row of P_{mt} to not be a scalar multiple of the same row of P_{0t}.

reflecting competition with an outside good. We focus on cost shocks, looking to falsify a model based on the pass-through of c_{Ut} to the retail price p_{Dt} and of c_{Dt} to the wholesale price p_{Ut}.

Suppose we can observe the per-unit wholesale price, but cannot observe lump-sum transfers between the two firms and don't know whether they are feasible. In either case, we assume the upstream firm moves first, setting the wholesale price, and the downstream firm moves second. We want to know whether we can falsify a model in which lump-sum transfers are feasible against a model where they are not, and vice versa.

Result 4. *In the vertical monopoly setting, if the truth is linear wholesale pricing, two-part tariffs can be falsified with cost side instruments, and vice versa.*

If lump-sum transfers are feasible, the upstream firm will set p_{Ut} equal to its marginal cost c_{Ut}, to avoid distortions away from the monopoly price in the downstream market, and will extract all profits by demanding a lump-sum payment equal to downstream monopoly profits. Given wholesale price p_{Ut} and downstream marginal cost c_{Dt}, the retailer will then set p_{Dt} to maximize downstream profits, solving

$$\max_{p_{Dt}}(p_{Dt} - c_{Dt} - p_{Ut})s_t \qquad \longrightarrow \qquad p_{Dt} - c_{Dt} \quad = \quad p_{Ut} + \frac{1}{\alpha(1-s_t)}$$

Instead of writing everything in terms of markups, it's helpful here to think of how we recover marginal costs from observables. Letting the $2P$ subscript indicate the two-part tariff model, we can calculate implied marginal costs with lump-sum payments, the inverse pass-through matrix, and the pass-through matrix as

$$\begin{bmatrix} c_{2PUt} \\ c_{2PDt} \end{bmatrix} = \begin{bmatrix} p_{Ut} \\ p_{Dt} - p_{Ut} - \frac{1}{\alpha(1-s_t)} \end{bmatrix}, \quad P_{2Pt}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & \frac{1}{1-s_t} \end{bmatrix}, \quad P_{2Pt} = \begin{bmatrix} 1 & 0 \\ 1-s_t & 1-s_t \end{bmatrix}$$

(Note that since the downstream price p_{Dt} maximizes joint profits, it responds identically to an increase in either the upstream or the downstream marginal cost; and since the wholesale price p_{Ut} is simply c_{Ut}, it does not respond to a change in the downstream marginal cost.)

What if lump-sum payments are not allowed? The upstream firm will set wholesale price $p_{Ut} > c_{Ut}$, after which the downstream firm solves

$$\max_{p_{Dt}}(p_{Dt} - c_{Dt} - p_{Ut})s_t(p_{Dt}) \qquad \longrightarrow \qquad p_{Dt} - c_{Dt} - p_{Ut} \quad = \quad \frac{1}{\alpha(1-s_t)}$$

The upstream firm sets the wholesale price to solve

$$\max_{p_{Ut}}(p_{Ut}-c_{Ut})s_t(p_{Dt}(p_{Ut}))$$

with first-order condition

$$s_t+(p_{Ut}-c_{Ut})\frac{\partial s_t}{\partial p_{Dt}}\frac{\mathrm{d}p_{Dt}}{\mathrm{d}p_{Ut}} \quad = \quad 0$$

We can recover $\frac{\mathrm{d}p_{Dt}}{\mathrm{d}p_{Ut}}$ from the downstream firm's first-order condition and the Implicit Function Theorem, getting $\frac{\mathrm{d}p_{Dt}}{\mathrm{d}p_{Ut}}=1-s_t$. Letting the L subscript indicate the linear pricing model, we can calculate imputed marginal costs, the inverse pass-through matrix, and the pass-through matrix as

$$\begin{bmatrix} c_{Ut} \\ c_{Dt} \end{bmatrix}=\begin{bmatrix} p_{Ut}-\frac{1}{\alpha(1-s_t)^2} \\ p_{Dt}-p_{Ut}-\frac{1}{\alpha(1-s_t)} \end{bmatrix}, P_{Lt}^{-1}=\begin{bmatrix} 1 & \frac{2s_t}{(1-s_t)^2} \\ -1 & \frac{1}{1-s_t} \end{bmatrix}, P_{Lt}=\frac{(1-s_t)^2}{1+s_t}\begin{bmatrix} \frac{1}{1-s_t} & -\frac{2s_t}{(1-s_t)^2} \\ 1 & 1 \end{bmatrix}$$

We can then calculate

$$P_{2Pt}^{-1}P_{Lt} \quad = \quad \begin{bmatrix} \frac{1-s_t}{1+s_t} & -\frac{2s_t}{1+s_t} \\ 0 & 1 \end{bmatrix}$$

Note the top-right off-diagonal is always negative, so nonzero in expectation: if we observe the wholesale price and have access to an instrument for downstream marginal cost, we can falsify two-part tariffs. In fact, this is obvious: if two-part tariffs were available, the wholesale price should be equal to the upstream marginal cost, and therefore not affected by downstream marginal cost. With linear pricing, however, there is nonzero pass-through of downstream cost to wholesale price. So if the truth is linear pricing, we would observe the wholesale price responding to the retailer's cost shifter, falsifying the two-part tariff model. Similarly, $P_{Lt}^{-1}P_{2Pt}$ has a positive top-right off-diagonal term, so if the true model was two-part tariffs, we can likewise falsify linear wholesale pricing.

However, note that while the top-right term of $P_{2Pt}^{-1}P_{Lt}$ is always negative, the bottom-left term is zero. Once the wholesale price p_{Ut} has been set, the downstream firm reacts to it in the same way in both models, playing the static best-response to it; the two models differ only in how p_{Ut} is determined. The two models still have different values of $[P_m]_{21}$ – the absolute pass-through of the upstream firm's *costs* to the retail price – because an increase in c_{Ut} affects the *wholesale price* differently under the two models. But because the downstream firm's objective function is the same in both models, the equilibrium response of the retail price *relative to* the response of the wholesale price is the same in the two models when upstream cost changes ($\frac{P_{mt21}}{P_{mt11}} = \frac{P_{0t21}}{P_{0t11}}$ in the notation of the previous example). This means that even under the wrong model, a change in upstream costs does not lead to a change in inferred downstream costs. (The same effect occurs for the second-mover firm if we want to falsify sequential competition a la Stackelberg when the truth is simultaneous competition (or vice versa); we explore this example in Appendix D.) This illustrates the need in Corollary 1 for shifters of all products' costs to be available. In the vertical monopoly example, if we could *only* vary the upstream producer's marginal cost and observe the effect this had on the retail price, we could not distinguish between the two models.[16]

4.2 Various Reasons Falsifiability Can Fail

The observation that upstream cost shifters alone can't falsify the two-part tariff model under true linear pricing (or vice versa) motivates us to return to the two-product duopoly setting and examine models where falsifiability by one or both types of instruments fails.

Rival costs don't work when $P_m^{-1}P_0$ is diagonal

Corollary 1 established that falsification by rival cost shifters requires the conditional expected value of $P_{mt}^{-1}P_{0t}$ to not be diagonal. In Section 2, we explored one example where $P_{mt}^{-1}P_{0t}$ was diagonal:

Result 5. *If the true model is competition in quantities, perfect competition can only be falsified by demand side instruments.*

Since markups are zero under perfect competition, $P_{PCt} = I$, so for any true model, $P_{PCt}^{-1}P_{0t} = P_{0t}$. Under Cournot competition, pass-through P_{Ct} is diagonal, but with diagonal terms less than 1 (and therefore not 1 in expectation). Cournot competition, like perfect competition, predicts no pass-through of rival costs, so rival cost shifters cannot distinguish between the two models. On the other hand, Cournot competition predicts that a change in a firm's own product characteristics will change price, while perfect competition does not, allowing own product characteristics excluded from price to falsify the wrong model.

Corollaries 1 and 2 don't hold when Assumption 6 is violated

Next, consider a class of models where the markups take the form

$$\Delta_{mj} = \Delta_j(s_t, p_t) + (1 - \lambda_{mj})c_{0jt},$$

[16] If we do not observe wholesale prices, we could form restrictions based on $c_{Ut} + c_{Dt}$ as in Villas-Boas (2007). To falsify either vertical model with cost side instruments we would need more than one product in the market; variation in a rival's upstream or downstream cost would then falsify the wrong model.

where $\Delta_j(s_t, p_t)$ is a function that is not model-specific. Note that this violates Assumption 6, since costs enter markups directly. Markups of this form arise in the model of collusion considered in Harrington (2022), where firms collude via cost coordination. These markups also arise in settings where two firms compete a la Bertrand in prices, but each one maximizes a weighted sum of profits and revenues, choosing price p_{jt} to solve

$$\max_{p_{jt}} \left\{\lambda_j(p_{jt} - c_{jt})s_{jt}(\cdot) + (1-\lambda_j)p_{jt}s_{jt}(\cdot)\right\} \quad = \quad \max_{p_{jt}} \left\{(p_{jt} - \lambda_j c_{jt})s_{jt}(\cdot)\right\}$$

This could occur, for example, if at least one of the firms hoped to be acquired, and the acquirer's valuation was likely to be based on revenue growth rather than profitability; this could also be a reduced-form for a firm maximizing profits subject to a revenue constraint, or maximizing revenues subject to a profit constraint, with the weights coming from the Lagrange multiplier in the maximization problem (Baumol, 1958). For simplicity, we put no market subscript on each firm's weight λ_i on profits, but one could be easily incorporated.

Result 6. *Suppose the true model is price competition with $\lambda_0 = (\lambda_{01}, \lambda_{02})$. The same model with misspecified weights $\lambda_m \neq \lambda_0$ cannot be falsified with cost side or with demand side instruments.*

Firm j's problem in this class of models is exactly the same as that of a firm maximizing profits, but with marginal costs equal to $\lambda_j c_{jt}$ instead of c_{jt}. Given that, it's unsurprising that the pass-through and inverse pass-through matrices are

$$P_{mt} \quad = \quad P_{Bt}\begin{bmatrix} \lambda_{m1} & 0 \\ 0 & \lambda_{m2} \end{bmatrix} \qquad \text{and} \qquad P_{mt}^{-1} \quad = \quad \begin{bmatrix} \frac{1}{\lambda_{m1}} & 0 \\ 0 & \frac{1}{\lambda_{m2}} \end{bmatrix} P_{Bt}^{-1}$$

where P_{Bt} is the pass-through matrix for ordinary profit-maximizing price competition. If the true model has weights $\lambda_0 = (\lambda_{01}, \lambda_{02})$ on profits relative to revenues, and an alternative model uses weights $\lambda_m = (\lambda_{m1}, \lambda_{m2})$, then

$$P_{mt}^{-1} P_{0t} \quad = \quad \begin{bmatrix} \frac{\lambda_{01}}{\lambda_{m1}} & 0 \\ 0 & \frac{\lambda_{02}}{\lambda_{m2}} \end{bmatrix}$$

This matrix is diagonal, so we do not expect cost side instruments to work. (Like in the vertical example, the misspecified model is wrong about how a shock to one firm's cost affects that firm's equilibrium price, but right about how the other firm responds to a change in rival's price, so rival's cost shocks cannot falsify the wrong model.)

Why demand side instruments don't work is more subtle, since $P_{mt}^{-1} P_{0t} \neq I$ when $\lambda_m \neq \lambda_0$. However, notice we can derive the total effect of the instrument $z_{jt}^{(k)}$ on the markups implied by any model m as $\frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z^{(k)}_{jt}} = \frac{\mathrm{d}\Delta_{jt}}{\mathrm{d}z^{(k)}_{jt}}$, which is the same for all models in this class, including the true model. As the only part of markups that is model-specific is a rescaling of c_{0jt}, and since product characteristics are excluded from cost, they cannot falsify any model m.

Product characteristics don't work if they don't move markups

Consider a class of models of "cost-plus pricing," where each firm sets its markup as a fixed fraction of marginal costs, $p_{jt} - c_{jt} = \phi_j c_{jt}$. (Once again, this violates Assumption 6.) This leads to the pass-through matrix

$$P_{mt} \quad = \quad \begin{bmatrix} 1+\phi_{m1} & 0 \\ 0 & 1+\phi_{m2} \end{bmatrix}$$

Suppose the true model is characterized by markup ratios $\phi_0 = (\phi_{01}, \phi_{02})$, and consider the problem of falsifying a misspecified model with markup ratios $\phi_m = (\phi_{m1}, \phi_{m2}) \neq \phi_0$. Once again, neither type of instrument will work:

Result 7. *If the true model is cost-plus pricing, a model with misspecified markup ratios cannot be falsified with either cost or demand side instruments.*

The first part should be clear: all models in this class predict zero pass-through of rival's costs, so cost side instruments shouldn't have any effect. Demand side instruments don't work in this class of models because a firm's markup only depends on its own cost, not on the characteristics of its product that are independent of cost. Thus, despite the fact that the matrix condition holds, demand side instruments which are excluded from cost cannot falsify model m, as these do not shift markups for any of the models in this class. In the language of Proposition 1, $P^{-1}_{mt} - P^{-1}_{0t}$ is nonzero, but $\frac{\partial p_0}{\partial z^{(k)}_{jt}}$ is zero because product characteristics don't shift equilibrium prices.

Instruments can only work if they shift prices

Consider a model where firm 2's pricing is determined nationally by the marketing department, which decides at the corporate level that a certain length of sandwich should cost \$5, or certain menu items should be 99 cents, with no relation to local cost or product characteristics. Like DellaVigna and Gentzkow (2019), we refer to such a pricing strategy as uniform pricing. Suppose that in the true model, firm 1 sets price p_{1t} separately in each market to maximize profits, while firm 2 uses uniform pricing; and suppose we wish to falsify a model where both firms use uniform pricing.

Result 8. *If in the true model, firm 1 maximizes profits in each market and firm 2 uses uniform pricing, a model where both firms use uniform pricing can be falsified by demand side instruments but not by cost side instruments.*

We can calculate the pass-through matrix for the Local-Uniform model where firm 1 prices locally[17] (indexed LU) and the Uniform-Uniform model[18] (indexed UU) as

$$P_{LUt} \quad = \quad \begin{bmatrix} 1 - s_{1t} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad P_{UUt} \quad = \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

With these two models, it is meaningless to consider conditions on $P_{mt}^{-1} P_{0t}$, since neither pass-through matrix is invertible. That said, it should be clear that neither model can be falsified using cost instruments if the other is the correct model, since both predict the same response (none) to any rival cost shock.

With product characteristic instruments, if the true model is LU, we should be able to falsify the UU model. Market-to-market differences in product 1's characteristics will lead to variation in observed price p_{1t} across markets, which would falsify the UU model.

Instruments don't work if nothing moves markups

Another simple class of models is one where firms set constant markups

$$p_{jt} \quad = \quad c_{jt} + \lambda_{mj}$$

In this setting, a misspecified model ($\lambda_m \neq \lambda_0$) cannot be falsified by either instrument.

Result 9. *If the true model is constant markups, a model of constant markups at a misspecified level cannot be falsified by cost or demand side instruments.*

Pass-through matrices are $P_{mt} = I$ for any model in this class. Moreover, markups are not a function of either cost or demand side instruments, so there can be no direct effect of any of these instruments. This again underscores the fact that falsifiability using instruments is about pass-through and not about the level of markups.

[17] Firm 2's costs don't affect p_{2t} and therefore don't affect p_{1t} either, and firm 1's costs don't affect p_{2t}. The only nonzero element of P_{0t} is therefore the upper left element, $\frac{\mathrm{d}p_{1t}}{\mathrm{d}c_{1t}}$, which we can calculate directly from firm 1's first-order condition using the Implicit Function Theorem.

[18] If both firms use uniform pricing, pass-through of local cost shocks is zero.

4.3 Beyond Two-Firm Simple Logit

Partly for expositional reasons but partly to give sharp results, the examples above focused on the case of two single-product firms facing simple logit demand. It's natural to ask how our results extend to more general settings.

In the examples above, falsifiability was either guaranteed or impossible. More generally, however, we distinguish between three cases, which we discuss for cost side instruments:

- *Case I: Falsifiability is guaranteed.* There are observables (w_t, z_t) for which the matrix $[P_m^{-1}P_0]^\star$ *cannot be* diagonal – for example, because certain off-diagonal elements of $P_{mt}^{-1}P_{0t}$ are always strictly positive (or always strictly negative) and therefore cannot have zero expected value.

- *Case II: Falsifiability is in some sense generic.* There are observables (w_t, z_t) for which $[P_m^{-1}P_0]^\star$ could in principle be diagonal, but won't typically be – for example, because some off-diagonal element of $P_{mt}^{-1}P_{0t}$ varies continuously and could take either sign, so its expectation being exactly zero requires a non-generic distribution of unobservables.

- *Case III: Falsifiability is impossible.* The matrix $P_{mt}^{-1}P_{0t}$ is always diagonal, so $[P_m^{-1}P_0]^\star$ is always diagonal.

The examples above focused on cases I and III, where a model either is always or is never falsifiable; in more complex settings, however, we expect case II to be the most common.

For an example of this, in a two-firm simple logit setting we consider a model of profit weights, but now with competition in prices rather than quantities. For simplicity, we'll suppose the two firms' profit weights are the same, so each firm j chooses price p_{jt} to solve

$$\max_{p_{jt}} \left\{ (p_{jt} - c_{jt}) s_{jt}(p_t) + \theta (p_{-jt} - c_{-jt}) s_{-jt}(p_t) \right\}$$

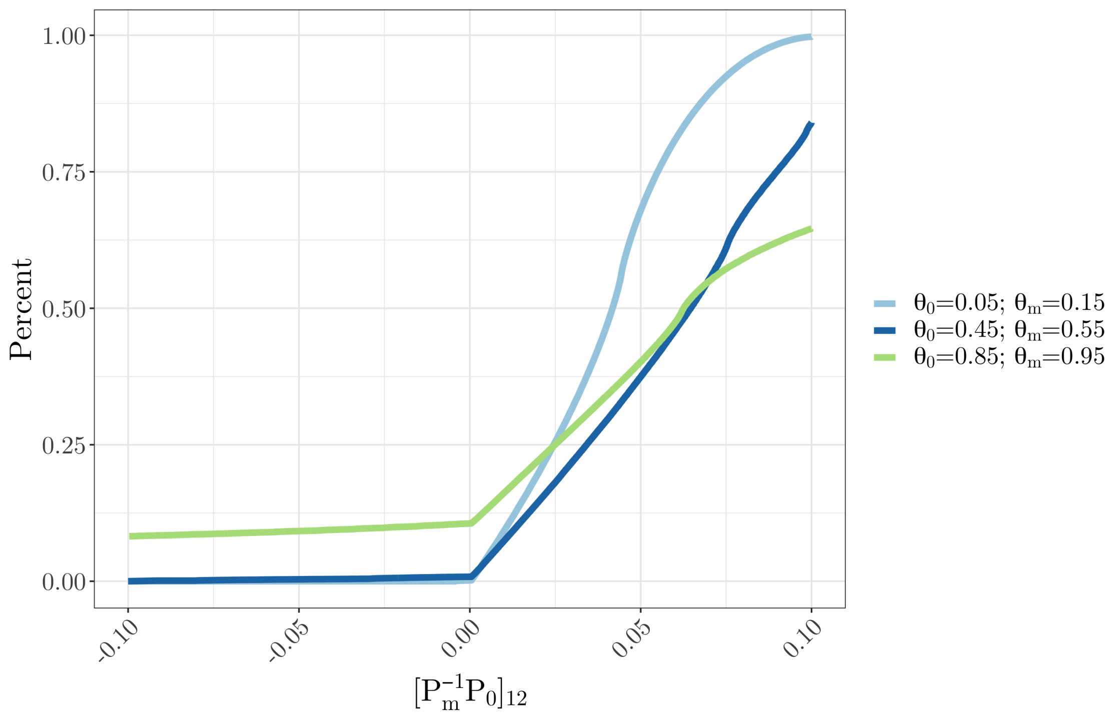
Under logit demand, the two firms' first-order conditions give the markup function

$$\Delta_{WBt} \quad = \quad \begin{bmatrix} \frac{1-(1-\theta)s_{2t}}{\alpha s_{0t} + \alpha(1-\theta^2) s_{1t} s_{2t}} \\ \\ \frac{1-(1-\theta)s_{1t}}{\alpha s_{0t} + \alpha(1-\theta^2) s_{1t} s_{2t}} \end{bmatrix}$$

with WB standing for the Weighted Bertrand model.

Falsifying a model where the weight on rival's profits is misspecified, $\theta_m \neq \theta_0$, is an instance of case II. For a given true value of θ_0 and a given misspecified model $\theta_m \neq \theta_0$, the off-diagonal terms of the matrix $P_m^{-1}P_0$ can be either positive or negative, but are not

FIGURE 1: CDF of $[P_{mt}^{-1}P_{0t}]_{12}$ for Betrand Profit Weight Models

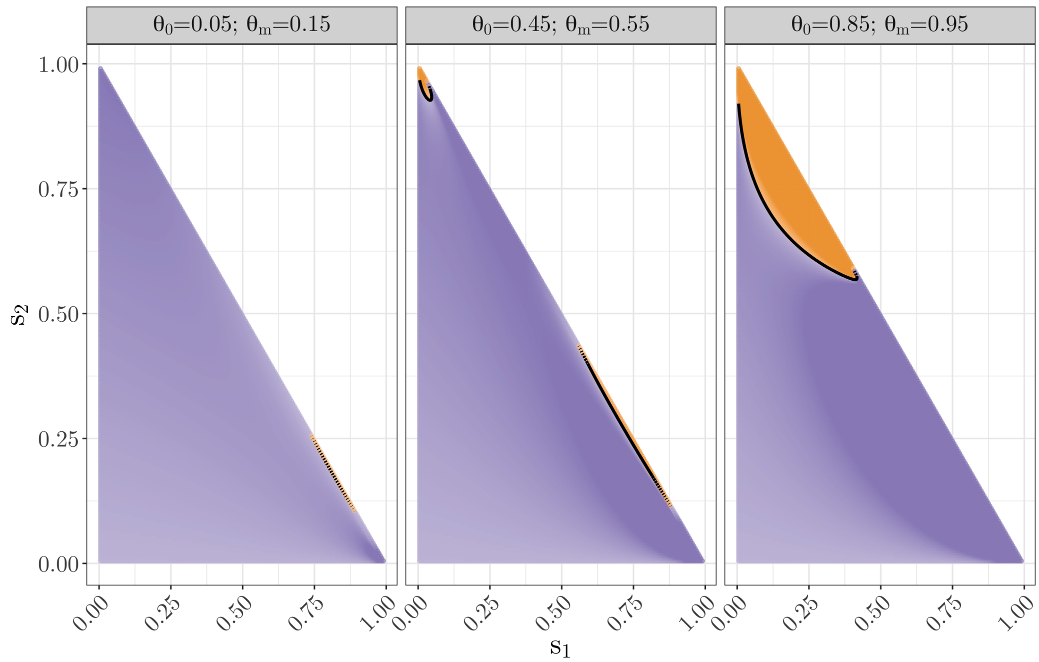


This figure plots distributions of $[P_{mt}^{-1}P_{0t}]_{12}$ over realizations of (s_{1t}, s_{2t}) for three different values of (θ_0, θ_m).

typically zero; and the diagonal terms can be greater than or less than 1, but are not typically equal to either 1 or to each other. We compute the top-right off-diagonal term, $[P_m^{-1}P_0]_{12}$, for a large sample of uniform draws of (s_{1t}, s_{2t}), and display the resulting distribution in Figure 1 for three representative pairs of true and misspecified models (θ_0, θ_m). While the distributions all have support that includes 0, the distributions are continuous, so getting a conditional expectation $E([P_{mt}^{-1}P_{0t}]_{12}) = 0$ would only hold for a non-generic set of observables.

We can also examine the value of the off-diagonal term at each point in the simplex of realizations of market shares (s_{1t}, s_{2t}). In each panel of Figure 2, we do so for three pairs of θ_0 and θ_m such that $\theta_m - \theta_0 = 0.1$. For each realization on the simplex, we indicate the value of $[P_m^{-1}P_0]_{12}$ with color gradients – darker orange indicates negative values that are larger in magnitude and darker purple indicates positive values that are larger in magnitude. The points where $[P_m^{-1}P_0]_{12} = 0$ are indicated in black. Immediately, one sees that for the vast majority of realizations of shares, $[P_m^{-1}P_0]_{12} > 0$. In fact, $[P_m^{-1}P_0]_{12}$ is always positive whenever the market share of both products is below 0.6. For an empirically relevant example, take the setting in Miller and Weinberg (2017). There, the outside option is defined in such a way that the market share of any product is less than 0.5 in all markets. In that case, the average of $[P_m^{-1}P_0]_{12}$ is positive in all three panels, and falsification of the wrong profit weight is possible with cost or demand side instruments.

FIGURE 2: Values of $[P_{mt}^{-1}P_{0t}]_{12}$ for Realizations of Market Shares in Bertrand Profit Weight Models



This figure plots magnitudes of $[P_{mt}^{-1}P_{0t}]_{12}$ over the simplex of (s_{1t}, s_{2t}) for three different values of (θ_0, θ_m). Darker purple (orange) shading indicates more positive (negative) values. Black indicates a zero value.

As we move away from logit demand, we expect the vast majority of cases to fall into case II – models that are typically falsifiable even if they can't be proven to always be. We offer one more example to support this intuition. Under Proposition 1, putting aside the direct effect, falsifiability depends on two conditions: the inverse pass-through matrices P_{mt}^{-1} and P_{0t}^{-1} being different, and their difference not being orthogonal to the effect of the instruments on equilibrium outcomes. Under Assumption 8, this comes down to $P_{mt}^{-1}P_{0t} \neq I$ in expectation. Still in the realm of two single-product firms but now with a general demand system we can show that it would be "difficult" for the Bertrand and Cournot models to satisfy $P_{mt}^{-1}P_{0t} = I$ in a particular market, and impossible under many commonly-used demand systems:

Result 10. *For a model with two single-product firms and general demand system $s_t = \mathcal{s}(p_t)$, if $P_{Bt}^{-1}P_{Ct} = I$, then*

$$\frac{\mathrm{d}}{\mathrm{d}p_t}\left(\frac{\partial \log s_{1t}}{\partial p_{1t}} \Big/ \frac{\partial \log s_{2t}}{\partial p_{2t}}\right)$$

must be equal to 0 *at market t equilibrium prices p_t.*

In many well-known demand systems, $\frac{\partial \log s_{1t}}{\partial p_{1t}}$ and $\frac{\partial \log s_{2t}}{\partial p_{2t}}$ move in opposite directions in response to price changes, making this condition impossible to satisfy. Quint (2014) catalogs several commonly-used demand systems under which demand is log-concave and log-supermodular. In that case, $-\frac{\mathrm{d}\log s_{1t}}{\mathrm{d}p_{1t}}$ would be positive and increasing in p_{1t}, while $-\frac{\mathrm{d}\log s_{2t}}{\mathrm{d}p_{2t}}$

would be positive and decreasing in p_{1t}, so $\frac{\partial \log s_{1t}}{\partial p_{1t}} \Big/ \frac{\partial \log s_{2t}}{\partial p_{2t}}$ would be increasing in p_1 (and by the same token decreasing in p_2). Quint (2014) also mentions numerical simulations showing that some other demand systems will yield the opposite – log-convex and log-submodular demand – in which case $-\frac{\mathrm{d} \log s_{1t}}{\mathrm{d} p_{1t}}$ would be decreasing in p_{1t} and $-\frac{\mathrm{d} \log s_{2t}}{\mathrm{d} p_{2t}}$ increasing. While there are certainly likely to be demand systems for which neither of these two cases hold, this example does seem to imply that $P_{Bt}^{-1} P_{Ct} = I$ would be a difficult result to achieve in a single market t; if it were to hold in expectation, then, this would be a knife-edge result based on the distribution of primitives across markets.[19]

5 Extension to Non-constant Marginal Cost

Everything up to now has assumed that a firm's marginal costs c_{0jt} are constant – or rather, vary only with cost shifters w_{jt}. Next, we consider what happens when cost may depend on quantity. As before, we begin with an extended example to build intuition, then formalize the results in a general setting.

5.1 Illustrative Example

We return to the environment from Sections 2 and 4 – two single-product firms facing simple logit demand in the absence of unobserved shocks – but this time with marginal costs which are linearly increasing in the quantity produced, $c_{0jt} = \gamma q_{jt} + \mathrm{w}_{jt}\tau$.

Suppose that the true model generating the data is perfect competition, and we wish to falsify the Cournot model. Recall from Section 4.1 that Cournot markups are $\Delta_{Cjt} = \frac{1 - s_{-jt}}{\alpha(1 - s_{1t} - s_{2t})}$, while markups under the true model are zero. This means that the true model with costs $c_{0jt} = \gamma q_{jt} + \mathrm{w}_{jt}\tau$ is observationally equivalent to the Cournot model with costs[20]

$$c_{mt} \quad = \quad \begin{bmatrix} \gamma q_{1t} + \mathrm{w}_{1t}\tau - \frac{1 - s_{2t}}{\alpha(1 - s_{1t} - s_{2t})} \\ \gamma q_{2t} + \mathrm{w}_{2t}\tau - \frac{1 - s_{1t}}{\alpha(1 - s_{1t} - s_{2t})} \end{bmatrix}. \tag{5}$$

This suggests a more general observation: models are only falsifiable once we restrict the marginal cost function. In the previous sections, we imposed in Assumption 3 that c_{mjt} could only depend on w_{jt}, not q_{jt}. In this section, we instead maintain that c_{0jt} and c_{mjt} can depend in an unrestricted way on q_{jt} and w_{jt}, but not on anything else, as in Assumption 2.

[19] Note also that this condition is not sufficient for $P_{Bt}^{-1} P_{Ct} = I$, only necessary; another condition must hold as well, and with J products, it would be J separate conditions that must hold.

[20] That is, a given set of primitives (x_t, w_t) would yield the same observable outcomes (p_t, s_t) under perfect competition with marginal costs c_{0jt} and under Cournot competition with costs given by Equation (5).

Given that assumption, the marginal cost function in Equation (5), which could "Cournot-rationalize" any data generated by the true model, is not admissible. Thus, with the right variation, we can hope to detect that the marginal costs implied by the Cournot model *must* depend on the other firm's quantity, and thus falsify the Cournot model.[21]

One instrument: rival cost shifter

To see how this works, we begin by considering the effect of a cost side instrument, $z_{jt} = \mathrm{w}^{(1)}_{-jt}$. Under the true model of perfect competition, equilibrium prices can be thought of as the solutions to the equation

$$F(\cdot) \quad \equiv \quad \begin{bmatrix} p_{1t} - c_{01t} \\ p_{2t} - c_{02t} \end{bmatrix} \quad = \quad \begin{bmatrix} p_{1t} - \mathrm{w}_{1t}\tau - \gamma s_{1t} \\ p_{2t} - \mathrm{w}_{2t}\tau - \gamma s_{2t} \end{bmatrix} \quad = \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

and we can calculate the marginal effect of a change in a cost shifter $\mathrm{w}^{(1)}_t$ on observable prices under the true model via the implicit function theorem,

$$\frac{\mathrm{d}p_t}{\mathrm{dw}^{(1)}_t} \quad = \quad - \left[\frac{\mathrm{d}F}{\mathrm{d}p_t}\right]^{-1} \left[\frac{\mathrm{d}F}{\mathrm{dw}^{(1)}_t}\right] \quad = \quad \frac{\tau^{(1)}}{D} \begin{bmatrix} 1 + \gamma\alpha s_{2t}(1 - s_{2t}) & \gamma\alpha s_{1t}s_{2t} \\ \gamma\alpha s_{1t}s_{2t} & 1 + \gamma\alpha s_{1t}(1 - s_{1t}) \end{bmatrix}$$

where $D = (1 + \gamma\alpha s_{1t}(1 - s_{1t}))(1 + \gamma\alpha s_{2t}(1 - s_{2t})) - \gamma^2\alpha^2 s^2_{1t}s^2_{2t}$. Note that even with perfect competition, there's positive pass-through of rival's costs: when firm 2's costs go up and it raises its price, firm 1's market share increases, increasing 1's marginal cost and therefore firm 1's price.

Having calculated the (true) effect of cost shifters on realized prices, we can calculate the effect of $\mathrm{w}^{(1)}_{2t}$ on firm 1's marginal costs inferred under the Cournot model, c_{m1t}, as

$$\frac{\mathrm{d}}{\mathrm{dw}^{(1)}_{2t}} \left(p_{1t} - \frac{1 - s_{2t}}{\alpha(1 - s_{1t} - s_{2t})}\right) \quad = \quad \frac{\tau^{(1)}}{D} \left[\frac{\gamma\alpha s_{1t}s_{2t}(1 - s_{2t})}{1 - s_{1t} - s_{2t}}\right].$$

[21]Interestingly, if we were seeking to falsify the Bertrand model, under logit demand with single-product firms, markups $\Delta_{Bjt} = \frac{1}{\alpha(1-s_{jt})}$ are solely a function of own market share, so the misspecified cost function implied by Bertrand competition, $c_{Bjt} = \gamma q_{jt} + \mathrm{w}_{jt}\tau - \frac{1}{\alpha(1-s_{jt})}$, does not violate the exclusion restriction and the model cannot be falsified by *any* instruments. However, we see this as very much a knife-edge case: with multi-product firms, even simple logit markups would depend on the market shares of the same firm's other products.

We can also calculate the true effect $\mathrm{w}_{2t}^{(1)}$ will have on firm 1's observed market share,

$$\frac{\mathrm{d}s_{1t}}{\mathrm{dw}_{2t}^{(1)}} \quad = \quad \frac{\partial s_{1t}}{\partial p_{1t}}\frac{\mathrm{d}p_{1t}}{\mathrm{dw}_{2t}^{(1)}} + \frac{\partial s_{1t}}{\partial p_{2t}}\frac{\mathrm{d}p_{2t}}{\mathrm{dw}_{2t}^{(1)}} \quad = \quad \frac{\tau^{(1)}}{D}\alpha s_{1t}s_{2t} \ .$$

By assumption, w_{2t} does not enter c_{1t} directly; so to not falsify the model, the change in c_{m1t} needs to be fully attributable to the change in firm 1's market share,

$$\frac{\partial c_{m1t}}{\partial s_{1t}}\frac{\mathrm{d}s_{1t}}{\mathrm{dw}_{2t}^{(1)}} \quad = \quad \frac{\mathrm{d}}{\mathrm{dw}_{2t}^{(1)}}\left(p_{1t} - \frac{1-s_{2t}}{\alpha(1-s_{1t}-s_{2t})}\right)$$

Given our calculations above, this requires

$$\frac{\partial c_{m1t}}{\partial s_{1t}} \quad = \quad \gamma\left(1 + \frac{s_{1t}}{1-s_{1t}-s_{2t}}\right) \tag{7}$$

So, if data is generated under perfect competition (with linear marginal costs), fitting the Cournot model to the data would require a marginal cost function c_{mt} satisfying Equation (7).

In terms of falsifying the wrong model, this is at least partially good news, in the sense that c_{m1t} depends on s_{2t}, violating the exclusion restriction. However, variation in $\mathrm{w}_{2t}^{(1)}$ alone may not be enough to detect it: if $\mathrm{w}_{2t}^{(1)}$ alone were changing, the data would trace a one-dimensional curve through (s_{1t}, s_{2t})-space, and we would not be able to tell that c_{m1t} depended directly on s_{2t} and not just on s_{1t}.[22]

Second instrument: own product characteristic

Next, we consider an additional instrument – this time, a characteristic of product 1, $x_{1t}^{(1)}$, which enters demand through the mean utility of product 1, $\delta_{1t} = x_{1t}\beta - \alpha p_{1t}$. While $x_{jt}^{(1)}$ does not enter costs directly, it still changes prices under the true model of perfect competition – by changing equilibrium market shares and therefore marginal costs. The effect of $x_{1t}^{(1)}$ on market shares is $\frac{\partial s_{1t}}{\partial x_{1t}^{(1)}} = \beta^{(1)}s_{1t}(1-s_{1t})$ and $\frac{\partial s_{2t}}{\partial x_{1t}^{(1)}} = -\beta^{(1)}s_{1t}s_{2t}$; we can calculate the true

[22] More formally, if $\mathrm{w}_{2t}^{(1)}$ were the only thing varying across markets, then observed market shares would be deterministic functions of $\mathrm{w}_{2t}^{(1)}$, $s_{jt} = \bar{s}_j(\mathrm{w}_{2t}^{(1)})$, and we could not rule out the Cournot model with the cost function

$$c_{mt} \quad = \quad \begin{bmatrix} \gamma q_{1t} + \mathrm{w}_{1t}\tau - \frac{1-\bar{s}_2(\bar{s}_1^{-1}(s_{1t}))}{\alpha(1-s_{1t}-\bar{s}_2(\bar{s}_1^{-1}(s_{1t})))} \\ \gamma q_{2t} + \mathrm{w}_{2t}\tau - \frac{1-\bar{s}_1(\bar{s}_2^{-1}(s_{2t}))}{\alpha(1-\bar{s}_1(\bar{s}_2^{-1}(s_{2t}))-s_{2t})} \end{bmatrix}$$

which does not violate the exclusion restriction.

effect of $x_{jt}^{(1)}$ on observed prices from Equation (6) using the implicit function theorem,

$$\frac{\mathrm{d}p_t}{\mathrm{d}x_t^{(1)}} = -\left[\frac{\mathrm{d}F}{\mathrm{d}p_t}\right]^{-1}\left[\frac{\mathrm{d}F}{\mathrm{d}x_t^{(1)}}\right] = \frac{\gamma\beta^{(1)}}{D}\begin{bmatrix} s_{1t}(1-s_{1t})+\gamma\alpha s_{1t}s_{2t}s_{0t} & -s_{1t}s_{2t} \\ -s_{1t}s_{2t} & s_{2t}(1-s_{2t})+\gamma\alpha s_{1t}s_{2t}s_{0t} \end{bmatrix}$$

where again $D = (1+\gamma\alpha s_{1t}(1-s_{1t}))(1+\gamma\alpha s_{2t}(1-s_{2t})) - \gamma^2\alpha^2 s_{1t}^2 s_{2t}^2$. (For intuition, recall that the true model is perfect competition with increasing marginal costs. When a change in x_{1t} increases δ_{1t}, product 1 gains market share and product 2 loses market share; product 1's cost goes up, product 2's cost goes down, so 1's price goes up and 2's goes down.)

Because $x_{1t}^{(1)}$ does not enter firm 1's marginal costs directly, to explain the data using the Cournot model, we would need the change in implied costs to be attributed to the observed change in market share,

$$\frac{\partial c_{m1t}}{\partial s_{1t}}\frac{\mathrm{d}s_{1t}}{\mathrm{d}x_{1t}^{(1)}} = \frac{\mathrm{d}}{\mathrm{d}x_{1t}^{(1)}}\left(p_{1t} - \frac{1-s_{2t}}{\alpha(1-s_{1t}-s_{2t})}\right)$$

Note that $x_{1t}^{(1)}$ affects s_{1t} both directly and through the change in prices. We can calculate

$$\frac{\mathrm{d}}{\mathrm{d}x_{1t}^{(1)}}\left(p_{1t} - \frac{1-s_{2t}}{\alpha s_{0t}}\right) = \frac{\gamma\beta^{(1)}}{D}\left(s_{1t}(1-s_{1t})+\gamma\alpha s_{1t}s_{2t}s_{0t}\right) - \frac{\gamma\beta^{(1)}}{D}\frac{s_{1t}}{s_{0t}}\left(\frac{1}{\gamma\alpha}+s_{2t}(1-s_{2t})\right)$$

and

$$\frac{\mathrm{d}s_{1t}}{\mathrm{d}x_{1t}^{(1)}} = \frac{\gamma\alpha\beta^{(1)}}{D}s_{1t}(1-s_{1t})\frac{1}{\alpha\gamma} + \frac{\gamma\alpha\beta^{(1)}}{D}s_{1t}s_{2t}s_{0t}$$

This means that to account for the change in inferred costs under the Cournot model, we would need

$$\frac{\partial c_{m1t}}{\partial s_{1t}} = \gamma\left(1 - \frac{s_{1t}}{s_{0t}}\frac{\frac{1}{\gamma\alpha}+s_{2t}(1-s_{2t})}{s_{1t}(1-s_{1t})+\gamma\alpha s_{1t}s_{2t}s_{0t}}\right) \tag{8}$$

Equations (7) and (8) should make it apparent that variation in a rival cost shifter and an own product characteristic instrument together suffice to falsify the Cournot model when the true model generating the data is perfect competition. From Equation (7), to rationalize changes in market outcomes when $\mathrm{w}_{2t}^{(1)}$ changes requires $\frac{\partial c_{m1t}}{\partial s_{1t}} > \gamma$; and from Equation (8), to rationalize changes in market outcomes when $x_{1t}^{(1)}$ changes requires $\frac{\partial c_{m1t}}{\partial s_{1t}} < \gamma$. Since no cost function can satisfy both, the model is falsified.

There is useful economic intuition for why $\frac{\partial c_{m1t}}{\partial s_{1t}}$ appears to be greater than γ (its true value) when $\mathrm{w}_{2t}^{(1)}$ changes but less than γ when $x_{1t}^{(1)}$ changes. In the former case, under the true model of perfect competition with increasing marginal costs, when $\mathrm{w}_{2t}^{(1)}$ goes up, firm 2 responds to the increase in costs by raising its price, reducing s_{2t}. Under the Cournot model, the decrease in s_{2t} reduces firm 1's markup function; so the Cournot model must attribute *more than 100%* of the increase in p_{1t} to an increase in marginal costs, since it predicts that firm 1's markup has shrunk. On the other hand, when $x_{1t}^{(1)}$ increases (assuming $\beta^{(1)}$ is positive), this results in increases in both s_{1t} and firm 1's markup under Cournot; so the Cournot model attributes part of the increase in p_{1t} to an increase in markup and part of it – less than 100% – to the increase in marginal costs due to increased production.

Next, we extend this intuition about what makes two instruments "different enough" to falsify a model to the general setting of multi-product firms facing an arbitrary demand system with unobserved shocks.

5.2 General Conditions for Falsification

Now consider the general demand system with J products, and the separable specification for the true marginal cost $c_{0jt} = \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) + \omega_{0jt}$ with $\bar{c}_{0j}$ differentiable. As in Assumption 2, this restricts the true cost function $\bar{c}_{0j}$ in two ways. First, we let a firm's cost only depend on its own quantities and cost shifters. As we know the true cost function cannot depend directly on rivals' quantities or on the instruments, we can restrict the function $\bar{c}_{mj}$ to have the same property. Second, we maintain a mean independence restriction of the unobservable cost shock with respect to the instruments. Under these conditions falsification is possible, as the example above illustrates.

To explore what is precisely required to falsify a model under these two restrictions on the cost function $\bar{c}_{mj}$, we return to Lemma 2, which we can restate as follows:

Lemma 3. *Suppose that Assumptions 1, 2, and 4 hold. Then, model m is falsified by the instruments z_{jt} if and only if for some j there exists no function $\bar{c}_{mj}$ such that for all k*

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = E\left[\left(\frac{\partial \bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt})}{\partial q_{jt}} - \frac{\partial \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt})}{\partial q_{jt}}\right)\frac{\mathrm{d}q_{jt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \quad a.s. \tag{9}$$

In the case of constant marginal costs, falsification depends only on whether the left hand side of Equation (9) is equal to zero: the restriction $\frac{\partial \bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt})}{\partial q_{jt}} = 0$ removes a degree of freedom by which unobserved costs can match differences in markups, making falsification easier. This leads to the straightforward conditions on pass-through matrices in Corollaries 1 and 2. With non-constant marginal costs, these conditions on pass-through are still relevant but no longer suffice on their own. With a single instrument, a cost function $\bar{c}_{mj}$ for a misspecified model $m \neq 0$ can typically be constructed to satisfy (9). This is easiest to see in the case of no unobservables, since we can often construct a cost function $\bar{c}_{mj}$ which satisfies (9) as a solution to the differential equation

$$\frac{\partial \bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt})}{\partial q_{jt}} = \frac{\partial \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt})}{\partial q_{jt}} + \left(\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}}(\mathrm{w}_{jt}, z_{jt}^{(k)}) - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}}(\mathrm{w}_{jt}, z_{jt}^{(k)}) \right) \Bigg/ \frac{\mathrm{d}q_{jt}}{\mathrm{d}z_{jt}^{(k)}}(\mathrm{w}_{jt}, z_{jt}^{(k)}) \tag{10}$$

This also suggests that with more than one instrument, falsification is likely to be impossible when the right-hand side of (10) is the same for all instruments. We formalize the added difficulty of falsifying a model under non-constant marginal cost in the next result:

Corollary 3. *Suppose that Assumptions 1, 2, and 4 hold, and that $P_{mt}^{-1}P_{0t} \neq I$ for all t. Suppose moreover that for the first instrument $k = 1$, there exists a cost function $\bar{c}_{mj}^{(1)}$ for each j satisfying Equation (9). A model of conduct m is not falsified by instruments z_{jt} if for each $k > 1$, there exists a constant ζ_k such that for all j and t,*

$$\frac{\mathrm{d}q_{jt}}{\mathrm{d}z_{jt}^{(k)}} = \zeta_k \frac{\mathrm{d}q_{jt}}{\mathrm{d}z_{jt}^{(1)}} \tag{11}$$

and

$$\left(\frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(k)}} \right) = \zeta_k \left(\frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(1)}} - \frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(1)}} \right). \tag{12}$$

Corollary 3 makes clear that, for multiple instruments to help, they must have different economic effects. For multiple instruments to falsify a model where a single instrument could not, the additional instruments must have differential effects on either quantities or on markups. In a standard differentiated product environment, it should be easy to select instruments which violate the conditions of Corollary 3, so falsification should typically be possible. For example, for many conduct models, markups depend directly on product characteristics. Thus, using a cost side instrument (which has no direct effect) and a demand side instrument would violate Equation (12). However, not every set of instruments will work. Under Assumption 7, falsification would fail if a researcher used two cost shifters from the same rival firm, as Equations (11) and (12) would both hold. Similarly, under Assumption 8, two characteristics of the same product do not permit falsification.

Corollary 3 is related to findings in Bresnahan (1982) and Lau (1982). They show that instruments that do not rotate demand do not permit falsification in a class of conduct parameter models when marginal costs are non-constant. This lack of falsification is not surprising, as Equations (11) and (12) hold in their context of homogenous products with a representative firm. Because there are no rivals, instruments are limited to the demand side and when the index restriction in Lau (1982) is satisfied, Equation (11) follows. Moreover, the markups for a model m can be expressed as $\Delta_{mjt} = \lambda_m \frac{\partial s_{jt}}{\partial q_{jt}}^{-1}$, so that $\Delta_{0jt} - \Delta_{mjt} = (\lambda_0 - \lambda_m)\frac{\partial s_{jt}^{-1}}{\partial q_{jt}}$. Thus, Equation (12) is violated only if the inverse demand is a non-linear function of z_{jt} (i.e., the instrument "rotates" demand).

5.3 "Failure Modes" with Non-constant Marginal Costs

The example in Section 5.1 illustrates that with non-constant marginal costs, one instrument is not enough to falsify the wrong model, but two instruments may be. Section 5.2 showed that two instruments can still fail to falsify the wrong model, but typically under strong restrictions on how they differ. We consider examples below to illustrate this point: even when pass-through matrices are different between model m and the truth, there are additional requirements for falsification to be guaranteed. We continue to work with the example from Section 5.1 – two single-product firms, simple logit demand, and linear marginal costs $c_{0jt} = \gamma q_{jt} + \mathrm{w}_{jt}\tau$.

Obviously redundant instruments

We do not expect to be able to falsify a model when all the instruments we use have identical effects. As one example, return to the question of falsifying Cournot competition when the truth is perfect competition, and suppose our two instruments were two different shifters of the same rival firm's costs, $z_{1t}^{(1)} = \mathrm{w}_{2t}^{(1)}$ and $z_{1t}^{(2)} = \mathrm{w}_{2t}^{(2)}$. (With non-constant marginal costs, the matrix $P_{Ct}^{-1}P_{PCt}$ is no longer diagonal.) Since both instruments affect market outcomes only by changing c_{02t}, their marginal effects on observed prices and market shares would be identical, simply rescaled by $\tau^{(2)}/\tau^{(1)}$. If a cost function would rationalize data under Cournot in the face of one rival cost shifter, it would still rationalize data in the face of a second shifter of the same rival's cost. (In the language of Corollary 3, (12) holds because rival cost shifters have no direct effect on markup so both sides are zero, and (11) holds with $\zeta_2 = \tau^{(2)}/\tau^{(1)}$ since the marginal effect of $\mathrm{w}_{2t}^{(k)}$ on market shares is simply $\tau^{(k)}$ times the effect of c_{2t}.)

Surprisingly redundant instruments

More surprisingly, perhaps, in the example in Section 5.1, a rival cost shifter $\mathrm{w}_{2t}^{(1)}$ together with a product characteristic $x_{2t}^{(1)}$ of the same rival would not suffice to falsify the Cournot model. As discussed earlier, under Assumptions 6 and 8, the effect of a product characteristic x_{jt} on a rival's markup is proportional to the effect of the same product's marginal cost. In this example, which satisfies both assumptions, the two instruments satisfy the two conditions of Corollary 3 and are therefore redundant.

Why these examples don't worry us

While we can come up with ways to satisfy Corollary 3 and therefore fail to falsify the wrong model with two instruments, we're confident that in most differentiated-product environments, this won't be a problem. Still looking to falsify Cournot competition when the true model is perfect competition, consider now the case of four or more single-product firms, with cost shifters as the instrument of choice. Focus on firms 1 and 2, and use as instruments the cost shifters of the other two firms, $z_{1t}^{(1)} = z_{2t}^{(1)} = \mathrm{w}_{3t}^{(1)}$ and $z_{1t}^{(2)} = z_{2t}^{(2)} = \mathrm{w}_{4t}^{(1)}$. With cost shifters, there are no direct effects on markup functions, so Equation (12) will hold; but we expect Equation (11) to typically be violated in sufficiently complex environments. Equation (11) (and therefore Corollary 3) would require

$$\frac{\mathrm{d}q_{1t}/\mathrm{dw}_{4t}^{(1)}}{\mathrm{d}q_{1t}/\mathrm{dw}_{3t}^{(1)}} \quad = \quad \frac{\mathrm{d}q_{2t}/\mathrm{dw}_{4t}^{(1)}}{\mathrm{d}q_{2t}/\mathrm{dw}_{3t}^{(1)}}$$

This might hold in a highly symmetric model such as non-nested logit, where substitution patterns are highly restricted. However, it should fail in any model general enough to admit a notion of "closer" and "more distant" substitutes. As an example, suppose products 2 and 4 are close substitutes – such as products within the same nest in a nested logit model – while no other pair of products is. We would then expect q_{2t} to respond strongly to a change in w_{4t} (operating through a change in p_{4t}), relative to its response to w_{3t}, while q_{1t} would not have this "stronger" response to w_{4t} relative to w_{3t}, which would violate this condition. Any model rich enough for some pairs of products to be "closer substitutes" than others should lead to violations of equation (11) in this way.

With product characteristics, we're free to use a firm's own product characteristic and a rival's, and these will typically not satisfy Equation (12). Under simple logit demand, a product characteristic of firm $j \neq 1$ has no direct effect on the markup function of firm 1.[23] This means that in the logit case, $\frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(k)}}$ will be nonzero when instrument k is a characteristic of product j and zero when it's a characteristic of a rival product, making (12) impossible to satisfy even with two firms. And even without this logit quirk, as long as a product characteristic's effect on the difference in markups $\Delta_{0jt} - \Delta_{mjt}$ is different for own-product versus rival-product characteristics, Equation (12) seems virtually impossible to satisfy.

While Corollary 3 is not an if-and-only-if result – failure of either Equation (11) or (12) does not guarantee a model is falsifiable – these conditions seem like the most natural way for falsifiability to fail. Satisfying Lemma 3 without satisfying the two conditions of Corollary

[23] Under Assumption 8, x_{jt} has similar effects to c_{jt}, and under the Cournot model with simple logit demand, rival cost pass-through is zero.

3 roughly requires $\frac{\mathrm{d}}{\mathrm{d}z_{jt}^{(k)}}(\Delta_{0jt} - \Delta_{mjt}) / \frac{\mathrm{d}q_{jt}}{\mathrm{d}z_{jt}^{(k)}}$ to be the same for all instruments in expectation *without* being the same realization-by-realization, which seems like it would require very "unlucky" distributions of unobservables. Thus, in settings with more than two firms, we expect rival cost shifters alone to give sufficient variation to typically falsify the wrong model if pass-throughs are sufficiently different; and even in settings with just two firms, we expect product characteristics to typically do likewise.

6 Conclusion

We discuss falsification of models of conduct in a general environment where researchers observe market outcomes for firms selling differentiated products. Our results highlight the economic features of different models that permit falsification, including the important role of cost pass-through. We also show that standard excluded instruments typically permit falsification in commonly used IO settings, both under constant and non-constant marginal costs.

The results in this paper give a new perspective on the foundations of empirical work that tests firm conduct. While testing conduct in finite samples requires additional statistical assumptions, fundamental economic differences are what ultimately distinguish models of conduct (see Appendix A). For applied researchers looking to use existing econometric procedures to evaluate a class of models of conduct, our framework allows them to determine ex ante which (if any) of the standard sources of variation in the data has the potential to distinguish the true model.

Moreover, our results provide a framework to propose and analyze new instruments and classes of models to be tested in applications. As one example, in ongoing work, Dearing, Magnolfi, and Sullivan (2022) use this framework to show that variation in ad-valorem taxes can falsify models of uniform versus local pricing, cost-plus pricing and constant markups models that cannot be falsified with standard cost or demand side instruments. We similarly anticipate that the results in this paper will offer a useful tool for researchers tackling new classes of models or instruments in a wide range of empirical settings.

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Appendix A Connection with Testing in Finite Sample

In the paper we discuss falsification of models of conduct. The population analysis that we develop, however, underpins the standard testing procedures used in empirical IO. To show that, we use the environment and notation of Duarte et al. (2022). In that environment, the falsifiable restriction in Lemma 1 implies a condition on the MSE of predicted markups, or

$$E\left[\left(\Delta^z_{0jt} - \Delta^z_{mjt}\right)^2\right] = 0,$$

where predicted markups for model m and the truth are constructed as:

$$\Delta^z_m = z\Gamma_m, \qquad \text{where } \Gamma_m = E\left[z'z\right]^{-1} E\left[z'\Delta_m\right],$$

and cost shifters are controlled for by orthogonalizing all variables with respect to w_{jt}. Note that, to implement the testability condition, the conditional moments of Lemma 1 are converted to unconditional moments.

The MSE in predicted markups can thus be used to measure lack-of-fit for model m when compared to the true model. Let $Q_m = E\left[\left(\Delta^z_{0jt} - \Delta^z_{mjt}\right)^2\right]$ denote the lack-of-fit. When $Q_m > 0$, model m is falsified. Duarte et al. (2022) show that we can also express Q_m as a GMM objective function:

$$Q_m = g_m' W g_m,$$

where $g_m = E[z_{jt}(p_{jt} - \Delta_{mjt})]$ and $W = E[z_{jt}z_{jt}']^{-1}$ is the weight matrix.

To operationalize a test for conduct,[24] researchers need to specify statistical hypotheses. Proposition 1 of Duarte et al. (2022) establishes that the statistical hypotheses that characterize the Rivers and Vuong (2002) (RV) test, as well as other tests used in the literature, can be expressed in terms of Q_m. The threat of misspecification suggests that researchers apply model selection tests, such as RV. For the RV test, the null hypothesis states that two competing models of conduct $m = 1, 2$ have the same lack-of-fit, or

$$H_0^{\text{RV}}: \; Q_1 = Q_2,$$

[24] In cases where models of conduct are nested, researchers have also pursued an estimation approach. Nevo (1998) and Magnolfi and Sullivan (2022) contrast testing versus estimation in this context.

while the alternatives correspond to cases of superior fit of one of the two models:

$$H_1^{\mathrm{RV}} : Q_1 < Q_2 \qquad \text{and} \qquad H_2^{\mathrm{RV}} : Q_2 < Q_1.$$

With this formulation of the hypotheses, the testing procedure determines which of the two models has the smallest lack of fit.

This model selection procedure naturally leverages falsification of either (or both) of the candidate models. When one of the two models is falsified (say, $m = 2$), and the other is not, this implies that $Q_2 > Q_1 = 0$, so that the RV test asymptotically rejects in favor of model 1. The findings in this paper on falsification of specific models thus help to understand the economic reasons behind the test results. For instance, consider the case of testing the two models of Bertrand ($m = 1$) and Cournot ($m = 2$), when the former is the true model, using cost side instruments. Because the pass-through matrices are different and the Bertrand pass-through matrix is not diagonal with logit demand, Cournot is falsified so that $Q_1 = 0$ and $Q_2 > 0$. The RV test asymptotically concludes for the true Bertrand model.

Importantly, when both models are not falsified, the RV test statistic is degenerate, causing severe inferential problems. Duarte et al. (2022) discuss this case and show that it corresponds to irrelevant instruments for testing. Our results help researchers prevent choices of instruments that may not achieve falsification, thus being irrelevant. For instance, suppose that the researcher wanted to test the two models of Cournot ($m = 1$) and perfect competition ($m = 2$), when the former is the true model, using cost side instruments under logit demand. In this case, perfect competition is not falsified, so that cost side instruments are irrelevant. Instead, the use of product characteristics as instruments would avoid degeneracy and allow the researcher to asymptotically conclude for the true model.

Appendix B Proofs

Proof of Lemma 1. As we note in the text, in our parametric framework, the falsifiable restriction in Equation (28) of Berry and Haile (2014) is[25]

$$E[\omega_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad E[p_{jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad 0 \qquad a.s.$$

Since observed prices are generated under the true model as

$$p_{jt} \quad = \quad \Delta_{0jt} + c_{0jt} \quad = \quad \Delta_{0jt} + \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt}$$

[25] See Section 6, Case 2 in Berry and Haile (2014) for a discussion of their non-parametric environment.

and $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ under Assumption 2, the falsifiable restriction is equivalent to

$$E[\Delta_{0jt} + \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad 0 \qquad a.s.$$

or equivalently

$$E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad \bar{c}_{mj}(\mathrm{w}_{jt}) - \bar{c}_{0j}(\mathrm{w}_{jt}) \qquad a.s.$$

giving the result. □

Proof of Lemma 2. We prove the inverse of both directions. If the model is not falsified, then there exists a set of cost functions $\{\bar{c}_{mj}(\mathrm{w}_{jt})\}_j$ satisfying the falsifiable restriction. Since neither $\bar{c}_{mj}$ nor $\bar{c}_{0j}$ can depend on the instruments, this means (by Lemma 1) that for each j and each value of w_{jt}, the expectation $E[\Delta_{0jt} - \Delta_{mjt} | \mathrm{w}_{jt}, z_{jt}]$ is almost everywhere constant with respect to z_{jt}. Taking the limit

$$\lim_{h \to 0} \frac{E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt} + h_k] - E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt}]}{h}$$

as in the text and noting that this must be 0 almost surely, this becomes

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = 0 \qquad a.s.$$

giving the result.

For the opposite direction, if for every j and k, $E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = 0$ almost surely, then $E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]$ must be the same for almost all values of z_{jt}. If so, define

$$\bar{c}_{mj}(\mathrm{w}_{jt}) \quad = \quad \bar{c}_{0j}(\mathrm{w}_{jt}) + E_{z_{jt}}\left[E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]\right]$$

and $\bar{c}_{mj}$ satisfy the equality condition in Lemma 1 almost surely, so the model is not falsified. □

Proof of Proposition 1. See text preceding Proposition 1. □

Proof of Corollary 1. Let $z_{jt}^{(k)}$, the k-th instrument for product j, be the i^{th} cost shifter of rival product ℓ, with $\tau^{(i)} \neq 0$. Since our instruments are cost shifters, under Assumption 6,

$\frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(k)}}$ and $\frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(k)}}$ are both 0. From Proposition 1, then, model m is falsified if for some (j,k),

$$E\left[\left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

Since the instrument $z_{jt}^{(k)}$ is a cost shifter of product $\ell \neq j$,

$$\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} = \frac{\partial p_0}{\partial c_t}\frac{\partial c_t}{\partial z_{jt}^{(k)}} = P_{0t} e_\ell \tau^{(i)},$$

where e_ℓ is the ℓ-th vector of the canonical basis. As a result, model m is falsified if

$$\tau^{(i)} E\left[\left(P_{mt}^{-1} P_{0t} - I\right)_j e_\ell \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

for some j and some $\ell \neq j$. Since by assumption $\tau^{(i)} \neq 0$, if we choose ℓ and j such that the (j,ℓ) element of $E[P_{mt}^{-1} P_{0t} \mid \mathrm{w}_{jt}, z_{jt}]$ is nonzero, this condition holds and the model is falsified. $\square$

Proof of Corollary 2. By Lemma 2, falsifiability comes down to whether for some j and k,

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

Let $z_{jt}^{(k)}$ be the i-th characteristic of product ℓ, and let $x_t^{(i)}$ denote the vector of that characteristic for all J products. Note that $x_t^{(i)}$ has both a direct effect on Δ_{mt} and an indirect effect through its impact on equilibrium prices,

$$\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d}x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} + \frac{\partial \Delta_{mt}}{\partial p_t}\frac{\mathrm{d}p_0}{\mathrm{d}x_t^{(i)}}$$

where $\frac{\mathrm{d}p_0}{\mathrm{d}x_t^{(i)}}$ is the effect of $x_t^{(i)}$ on equilibrium prices under the true model 0.

Under Assumption 8, $x_{jt}^{(i)}$ and p_{jt} affect Δ_{mt} and Δ_{0t} *directly* only through δ_{jt}, so

$$\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\frac{\partial \delta_t}{\partial x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\beta^{(i)} I$$

and

$$\frac{\partial \Delta_{mt}}{\partial p_t} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\frac{\partial \delta_t}{\partial p_t} = \frac{\partial \Delta_{mt}}{\partial \delta_t}(-\alpha I)$$

and, putting the two together,

$$\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha}\frac{\partial \Delta_{mt}}{\partial p_t}$$

We already defined the notation $H_{\Delta_{mt}} = \frac{\partial \Delta_{mt}}{\partial p_t}$, so $\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha} H_{\Delta_{mt}}$.

Next, to calculate $\frac{\mathrm{d} p_t}{\mathrm{d} x_t^{(i)}}$, recall that equilibrium prices are defined implicitly as the solution to the true first-order conditions $F(\cdot) = p_t - c_t - \Delta_{0t} = 0$. By the implicit function theorem,

$$\frac{\mathrm{d} p_t}{\mathrm{d} x_t^{(i)}} = -\left[\frac{\partial F}{\partial p_t}\right]^{-1}\left[\frac{\partial F}{\partial x_t^{(i)}}\right] = -\left[I - H_{\Delta_{0t}}\right]^{-1}\left[-\frac{\partial \Delta_{0t}}{\partial x_t^{(i)}}\right] = -\left[I - H_{\Delta_{0t}}\right]^{-1}\left[\frac{\beta^{(i)}}{\alpha} H_{\Delta_{0t}}\right]$$

Recalling that $P_{mt} = (I - H_{\Delta_{mt}})^{-1}$, this is

$$\frac{\mathrm{d} p_t}{\mathrm{d} x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha} P_{0t}\left(I - P_{0t}^{-1}\right) = \frac{\beta^{(i)}}{\alpha}\left(I - P_{0t}\right)$$

Plugging these into $\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d} x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} + \frac{\partial \Delta_{mt}}{\partial p_t}\frac{\mathrm{d} p_t}{\mathrm{d} x_t^{(i)}}$ gives

$$\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d} x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha} H_{\Delta_{mt}} + H_{\Delta_{mt}}\left(\frac{\beta^{(i)}}{\alpha}\left(I - P_{0t}\right)\right) = -\frac{\beta^{(i)}}{\alpha} H_{\Delta_{mt}} P_{0t} = -\frac{\beta^{(i)}}{\alpha}(I - P_{mt}^{-1}) P_{0t}$$

From this,

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d} z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d} z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = E\left[\frac{\beta^{(i)}}{\alpha}\left((P_{0t}^{-1} - P_{mt}^{-1}) P_{0t}\right)_{j,\ell} \mid \mathrm{w}_{jt}, z_{jt}\right]$$

$$= \frac{\beta^{(i)}}{\alpha} E\left[\left(I - P_{mt}^{-1} P_{0t}\right)_{j,\ell} \mid \mathrm{w}_{jt}, z_{jt}\right]$$

Thus, unless $E[(P_{mt}^{-1} P_{0t})_j \mid \mathrm{w}_{jt}, z_{jt}] = e_j'$ for each j almost surely, there is some (j,k) satisfying $E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d} z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d} z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0$ w.p.p., the condition for falsifiability under Lemma 2. □

Proof of Lemma 3. In our parametric framework, the falsifiable restriction in Equation (28) of Berry and Haile (2014) is that for all j there exists a cost function $\bar{c}_{mj}$ such that:

$$E[p_{jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = E[\bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \qquad a.s.$$

By plugging in for p_t as in the proof of Lemma 1, a model m is not falsified if for all j there exists a cost function $\bar{c}_{mj}$ such that

$$E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = E[\bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt}) - \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \qquad a.s.$$

The result thus follows by extending Lemma 2 to this restriction, so that a model is falsified if for some j there exists no cost function $\bar{c}_{mj}$ such that for all k

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = E\left[\frac{\mathrm{d}\bar{c}_{mj}(q_{jt}, \mathrm{w}_{jt})}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt})}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \qquad a.s.$$

□

Proof of Corollary 3. By assumption, we have that for all j there exists $\bar{c}_{mj}^{(1)}$ such that

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(1)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(1)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = E\left[\frac{\mathrm{d}\bar{c}_{mj}^{(1)}(q_{jt}, \mathrm{w}_{jt})}{\mathrm{d}z_{jt}^{(1)}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt})}{\mathrm{d}z_{jt}^{(1)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \qquad a.s.$$

Writing the total effect $\frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(1)}}$ of the instrument on each model's markups as the sum of the direct and indirect effects, and writing the indirect effects in terms of the change in equilibrium market shares rather than prices, gives

$$\frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(1)}} = \frac{\partial\Delta_{mt}}{\partial z_{jt}^{(1)}} + \frac{\partial\Delta_{mt}}{\partial p_t}\frac{\partial p_t}{\partial q_t}\frac{\mathrm{d}q_t}{\mathrm{d}z_{jt}^{(1)}}$$

From there, Equations (11) and (12) imply that for any k,

$$\begin{aligned}
\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} &= \frac{\partial\Delta_{0j}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{mj}}{\partial z_{jt}^{(k)}} + \left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\partial p_t}{\partial q_t}\frac{\mathrm{d}q_t}{\mathrm{d}z_{jt}^{(k)}} \\
&= \zeta_k \left(\frac{\partial\Delta_{0j}}{\partial z_{jt}^{(1)}} - \frac{\partial\Delta_{mj}}{\partial z_{jt}^{(1)}} + \left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\partial p_t}{\partial q_t}\frac{\mathrm{d}q_t}{\mathrm{d}z_{jt}^{(1)}}\right) \\
&= \zeta_k \left(\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(1)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(1)}}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\mathrm{d}\bar{c}^{(1)}_{mj}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(k)}_{jt}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(k)}_{jt}} &= \left(\frac{\mathrm{d}\bar{c}^{(1)}_{mj}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}q_{jt}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}q_{jt}}\right)\frac{\mathrm{d}q_{jt}}{\mathrm{d}z^{(k)}_{jt}} \\
&= \zeta_k\left(\frac{\mathrm{d}\bar{c}^{(1)}_{mj}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}q_{jt}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}q_{jt}}\right)\frac{\mathrm{d}q_{jt}}{\mathrm{d}z^{(1)}_{jt}} \\
&= \zeta_k\left(\frac{\mathrm{d}\bar{c}^{(1)}_{mj}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(1)}_{jt}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(1)}_{jt}}\right)
\end{aligned}$$

Therefore, we have that for any k and j, $\bar{c}^{(1)}_{mj}$ is such that

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z^{(k)}_{jt}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z^{(k)}_{jt}} \mid \mathrm{w}_{jt}, z_{jt}\right] = E\left[\frac{\mathrm{d}\bar{c}^{(1)}_{mj}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(k)}_{jt}} - \frac{\mathrm{d}\bar{c}_{0j}(q_{jt},\mathrm{w}_{jt})}{\mathrm{d}z^{(k)}_{jt}} \mid \mathrm{w}_{jt}, z_{jt}\right] \qquad a.s.$$

and by Lemma 3 the result follows. □

Appendix C Markup Assumption

Assumption 6 holds naturally for a wide range of models where firms choose actions to maximize profits. We suppress the market index t and suppose for simplicity that products $i = 1$ through f are sold by the same firm, and that the firm chooses a set of actions $\{a_i\}_{i=1}^{f}$ to maximize profits,

$$\max_{\{a_i\}, i\in f} \sum_{i\in f}(p_i(a) - c_i)s_i(a)$$

where prices $p(\cdot)$ and market shares $s(\cdot)$ are determined by the actions taken by all firms. First-order conditions are then

$$\sum_{i=1}^{f}\frac{\partial p_i}{\partial a_j}s_i + \sum_{i=1}^{f}(p_i - c_i)\frac{\partial s_i}{\partial a_j} \quad = \quad 0$$

or

$$\begin{bmatrix} \frac{\partial s_1}{\partial a_1} & \frac{\partial s_2}{\partial a_1} & \cdots & \frac{\partial s_f}{\partial a_1} \\ \\ \frac{\partial s_1}{\partial a_2} & \frac{\partial s_2}{\partial a_2} & \cdots & \frac{\partial s_f}{\partial a_2} \\ \\ \vdots & \vdots & \vdots & \vdots \\ \\ \frac{\partial s_1}{\partial a_f} & \frac{\partial s_2}{\partial a_f} & \cdots & \frac{\partial s_f}{\partial a_f} \end{bmatrix} \begin{bmatrix} p_1 - c_1 \\ \\ p_2 - c_2 \\ \\ \vdots \\ \\ p_f - c_f \end{bmatrix} \quad = \quad - \begin{bmatrix} \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_1} \\ \\ \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_2} \\ \\ \vdots \\ \\ \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_f} \end{bmatrix}$$

Stacking across firms, we then get

$$\left[\Omega \odot \left[\frac{\partial s}{\partial a} \right]' \right] \Delta \quad = \quad - \left[\Omega \odot \left[\frac{\partial p}{\partial a} \right]' \right] s$$

where Ω is the ownership matrix,[26] and therefore

$$\Delta \quad = \quad - \left[\Omega \odot \left[\frac{\partial s}{\partial a} \right]' \right]^{-1} \left[\Omega \odot \left[\frac{\partial p}{\partial a} \right]' \right] s$$

Note that the right-hand side has no room for costs or product characteristics to enter directly – it's all just ownership structure and the way that firm actions a map to market outcomes (p, s), which depends on the demand system.

(Within this more general model, Bertrand is just the special case where firms choose prices, so $p(a) = a$ and therefore $\frac{\partial p}{\partial a} = I$; and Cournot is the special case where firms choose quantities so $s(a) = a$ and $\frac{\partial s}{\partial a} = I$. Here we're being more general about what exactly firms are choosing, and therefore what exactly they're assuming other firms are holding fixed while they optimize.)

This assumption also holds if firms maximize any weighted sum of their own profits, other firms' profits, and consumer surplus (or total welfare). Suppose the firm selling product j maximizes

$$\sum_{i=1}^{J} \gamma_{ji}(p_i - c_i)s_i + \lambda_j CS$$

where γ_{ji} is the weight the firm puts on the profits from product i (whether or not i is one of the same firm's products) and CS is consumer surplus. The first-order condition with

[26] This is defined as $\Omega_{ij} = 1$ if products i and j are sold by the same firm, and zero otherwise.

respect to action a_j is then

$$\sum_{i=1}^{J}\gamma_{ji}\frac{\partial p_i}{\partial a_j}s_i + \sum_{i=1}^{J}\gamma_{ji}(p_i - c_i)\frac{\partial s_i}{\partial a_j} - \lambda_j s_j \quad = \quad 0$$

or, stacking and rearranging,

$$\Delta \quad = \quad \left[\Gamma \odot \left[\frac{\partial s}{\partial a}\right]'\right]^{-1}\left[\Lambda - \Gamma \odot \left[\frac{\partial p}{\partial a}\right]'\right] s$$

where Γ is a matrix of the γ_{ji} terms and Λ is a diagonal matrix of the λ_j terms. Once again, the right-hand side contains only constants and features of the demand system, not costs or product characteristics.

Finally, consider a market with some first-movers and some second-movers. To avoid getting bogged down in notation, we show the result for two single-product firms facing general demand, but the intuition is the same more generally. Conditional on the action a_1 chosen by the first-mover, the second-mover chooses a_2 to maximize $(p_2 - c_2)s_2$, giving first-order condition

$$(p_2 - c_2)\frac{\partial s_2}{\partial a_2} + \frac{\partial p_2}{\partial a_2}s_2 \quad = \quad 0$$

Defining $F(a_1, a_2)$ as the left-hand side, then, a_2 is implicitly defined as a function of a_1 as the solution to $F(a_1, a_2) = 0$, so by the implicit function theorem,

$$a_2'(a_1) \quad = \quad -\frac{\frac{\partial F}{\partial a_1}}{\frac{\partial F}{\partial a_2}} \quad = \quad -\frac{\frac{\partial p_2}{\partial a_1}\frac{\partial s_2}{\partial a_2} + (p_2 - c_2)\frac{\partial^2 s_2}{\partial a_2 \partial a_1} + \frac{\partial^2 p_2}{\partial a_1 \partial a_2}s_2 + \frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_1}}{\frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_2} + (p_2 - c_2)\frac{\partial^2 s_2}{\partial a_2^2} + \frac{\partial^2 p_2}{\partial a_2^2}s_2 + \frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_2}}$$

We can go a step further, rewriting firm 2's first-order condition as $p_2 - c_2 = -\frac{\partial p_2}{\partial a_2}s_2 \Big/ \frac{\partial s_2}{\partial a_2}$ and plugging that into the expression for a_2', to emphasize that a_2 depends only on features of the demand system (how p and s respond to a) and therefore not directly on costs. The first-mover's problem is

$$\max(p_1(a_1, a_2(a_1)) - c_1)s_1(a_1, a_2(a_1))$$

with first-order condition

$$\frac{\partial p_1}{\partial a_1}s_1 + \frac{\partial p_1}{\partial a_2}a_2' s_1 + (p_1 - c_1)\frac{\partial s_1}{\partial a_1} + (p_1 - c_1)\frac{\partial s_1}{\partial a_2}a_2' \quad = \quad 0$$

whence

$$p_1 - c_1 \quad = \quad -\frac{\frac{\partial p_1}{\partial a_1} s_1 + \frac{\partial p_1}{\partial a_2} a_2' s_1}{\frac{\partial s_1}{\partial a_1} + \frac{\partial s_1}{\partial a_2} a_2'}$$

We therefore have both firms' markups $p_j - c_j$ as functions of the demand system, with no place for marginal costs or product characteristics to enter directly. If we had infinite patience, we could make this same argument for the general model of many multi-product firms with some first- and some second-movers, and by induction, with more than two "rounds" of actions.

Appendix D Sequential versus Simultaneous Competition Example

Suppose that firms compete in prices, but in the true model, firm 1 moves first, committing to its price p_{1t} before firm 2 chooses its price. We'll label this model SB1, for Stackelberg-Bertrand with firm 1 as the first mover.

Result 11. *If the true model is sequential price competition with firm 1 moving first (SB1), then simultaneous price competition, and sequential price competition with firm 2 moving first, can both be falsified using either type of instruments.*

If the true model is simultaneous price competition, a model of sequential price competition with either firm moving first can be falsified with either type of instruments.

Once p_{1t} has been set, firm 2 takes it as fixed and maximizes $(p_{2t} - c_{2t})s_{2t}$, leading to markup $p_{2t} - c_{2t} = -\frac{s_{2t}}{\partial s_{2t}/\partial p_{2t}} = \frac{1}{\alpha(1-s_{2t})}$. Firm 1, however, accounts for firm 2's response to its own price, solving

$$\max_{p_{1t}} \, (p_{1t} - c_{1t}) \, s_{1t}(p_{1t}, p_{2t}(p_{1t}))$$

giving first-order condition

$$s_{1t} + (p_{1t} - c_{1t}) \left(\frac{\partial s_{1t}}{\partial p_{1t}} + \frac{\partial s_{1t}}{\partial p_{2t}} \frac{\mathrm{d} p_{2t}}{\mathrm{d} p_{1t}} \right) \quad = \quad 0$$

With $p_{2t}(p_{1t})$ defined implicitly as the solution to firm 2's first-order condition, we can calculate $\frac{\mathrm{d}p_{2t}}{\mathrm{d}p_{1t}}$ via the Implicit Function Theorem, and find that it simplifies to $\frac{\mathrm{d}p_{2t}}{\mathrm{d}p_{1t}} = \frac{s_{1t}s_{2t}}{1-s_{2t}}$, allowing us to write firm 1's first-order condition as

$$s_{1t} + (p_{1t} - c_{1t}) \left(-\alpha s_{1t}(1 - s_{1t}) + \alpha s_{1t} s_{2t} \frac{s_{1t} s_{2t}}{1 - s_{2t}} \right) \quad = \quad 0$$

and therefore, after some simplificiation, we can calculate the markup function as

$$\Delta_{SB1t} = \frac{1}{\alpha}\begin{bmatrix} \frac{1}{\frac{s_{0t}}{1-s_{2t}}+s_{1t}s_{2t}} \\ \frac{1}{1-s_{2t}} \end{bmatrix}$$

While the algebra gets messy, we can calculate

$$P_{SB1t}^{-1} = \begin{bmatrix} \frac{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2+s_{0t}s_{1t}-s_{1t}s_{2t}(1-s_{2t})^2(1-2s_{1t})}{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2} & \frac{-s_{1t}s_{2t}(1-s_{2t})^2(1-2s_{2t})}{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2} \\ -\frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{2t}} \end{bmatrix}$$

and, dropping a constant,

$$P_{SB1t} \propto \begin{bmatrix} \frac{1}{1-s_{2t}} & \frac{s_{1t}s_{2t}(1-s_{2t})^2(1-2s_{2t})}{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2} \\ \frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2+s_{0t}s_{1t}-s_{1t}s_{2t}(1-s_{2t})^2(1-2s_{1t})}{(s_{0t}+s_{1t}s_{2t}(1-s_{2t}))^2} \end{bmatrix}$$

Recalling from earlier that simultaneous price competition gives inverse pass-through

$$P_{Bt}^{-1} = \begin{bmatrix} \frac{1}{1-s_{1t}} & -\frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ -\frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{2t}} \end{bmatrix}$$

we can calculate the signs of off-diagonal terms of $P_{Bt}^{-1}P_{SB1t}$ as

$$P_{Bt}^{-1}P_{SB1t} = \begin{bmatrix} \star & - \\ 0 & \star \end{bmatrix}$$

Since the upper-right off-diagonal is always negative, it must be negative in expectation, giving the first result. We can similarly show that the upper-right off-diagonal of $P_{SB1t}^{-1}P_{Bt}$ is always positive, so we can falsify SB1 when the true model is Bertrand. And finally, we can calculate P_{SB2t}, the pass-through matrix when firm 2 is the first-mover, and show that the top-right offer-diagonal of $P_{SB1t}^{-1}P_{SB2t}$ is always positive, so we can falsify the model where the wrong player is first-mover.

For intuition, note first that the bottom-left off-diagonal term of $P_{Bt}^{-1}P_{SB1t}$ being zero is not a knife-edge fluke, but an economically meaningful result. As in the upstream-downstream example in the text, under either simultaneous *or* sequential competition with firm 1 moving first, firm 2 chooses a price which is a static best-response to the actual value of p_{1t}. As a result, the response of p_{2t} to a change in p_{1t} is the same under either model; the two differ only in how p_{1t} is determined. So while a shock to c_{1t} will affect both equilibrium prices differently in the two models, the marginal effect on p_{2t} *relative to* the effect on p_{1t} will be exactly the same. As a result, a shock to c_{1t} will not be interpreted as a change in c_{2t} under either model, so the wrong model cannot be falsified through that channel.

On the other hand, the top-right off-diagonal term of $P_{Bt}^{-1} P_{SB1t}$ is always negative. For most markets, the Stackelberg model, like simultaneous competition, predicts positive pass-through of c_2 to p_1; but sequential competition predicts a smaller pass-through of costs, relative to the pass-through of c_2 to p_2, than does simultaneous competition. Under Bertrand competition, an increase in c_{2t} increases p_{2t}, which increases p_{1t} because the two firms' prices are strategic complements. This happens as well with Stackelberg-Bertrand, but with an added effect: since $\frac{\mathrm{d}p_{2t}}{\mathrm{d}p_{1t}} = \frac{s_{1t}s_{2t}}{1-s_{2t}}$ is increasing in s_{2t}, the increase in c_{2t}, by increasing p_{2t}, decreases the sensitivity of p_{2t} to p_{1t}. This leads firm 1 to lower its price, since firm 2 will not respond as strongly as before. The net effect is that an increase in c_{2t} induces a smaller increase in p_{1t} under sequential competition. In the extreme case where $s_{2t} > \frac{1}{2}$, this dynamic effect (the reduction in $\frac{\mathrm{d}p_{2t}}{\mathrm{d}p_{1t}}$) dominates and firm 1 will *cut* its price when c_2 goes up. In that case, falsifying sequential competition from the effect of c_{02} on c_{m1} is most obvious – an increase in c_{02} leads to a decrease in p_1, which in a model of simultaneous competition could only be rationalized by an accompanying decrease in c_1.

We can also consider the model of sequential competition in quantities. Suppose again that firm 1 moves first. With s_{1t} set, firm 2 solves

$$\max_{s_{2t}} (p_{2t}(s_{1t}, s_{2t}) - c_{2t}) s_{2t}$$

with first-order condition $p_{2t} - c_{2t} + \frac{\partial p_{2t}}{\partial s_{2t}} s_{2t} = 0$, from which we can calculate $p_{2t} - c_{2t} = \frac{1}{\alpha}\frac{1-s_{1t}}{s_{0t}}$ as well as $\frac{\mathrm{d}s_{2t}}{\mathrm{d}s_{1t}} = -\frac{s_{2t}}{1-s_{1t}}$. Moving first, firm 1 solves

$$\max_{s_{1t}} (p_{1t}(s_{1t}, s_{2t}(s_{1t})) - c_{1t}) s_{1t}$$

giving first-order condition

$$s_{1t}\left(\frac{\partial p_{1t}}{\partial s_{1t}} + \frac{\partial p_{1t}}{\partial s_{2t}}\frac{\mathrm{d}s_{2t}}{\mathrm{d}s_{1t}}\right) + (p_{1t} - c_{1t}) \quad = \quad 0$$

Plugging in (using $\left[\frac{\partial p_t}{\partial s_t}\right] = \left[\frac{\partial s_t}{\partial p_t}\right]^{-1}$) and simplifying, we get $p_{1t} - c_{1t} = \frac{1}{\alpha(1-s_{1t})}$. From the markup function

$$\Delta_{SC1t} = \begin{bmatrix} \frac{1}{\alpha(1-s_{1t})} \\ \frac{1-s_{1t}}{\alpha s_{0t}} \end{bmatrix}$$

(with $m = SC1$ for Stackelberg-Cournot with firm 1 as the first mover), we then calculate

$$P_{SC1t}^{-1} = \begin{bmatrix} \frac{1}{1-s_{1t}} & -\frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ 0 & \frac{1-s_{1t}}{s_{0t}} \end{bmatrix} \quad \text{and} \quad P_{SC1t} = s_{0t} \begin{bmatrix} \frac{1-s_{1t}}{s_{0t}} & \frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ 0 & \frac{1}{1-s_{1t}} \end{bmatrix}$$

Since the pass-through and inverse pass-through matrices of simultaneous Cournot competition are both diagonal, this immediately gives us the top-right off-diagonal of $P_{SC1t}^{-1}P_{Ct}$ is negative and the top-right off-diagonal of $P_{Ct}^{-1}P_{SC1t}$ is positive. Using symmetry, we can easily calculate P_{SC2t}, the pass-through matrix if firm 2 is the first mover, and find that $P_{SC2t}^{-1}P_{SC1t}$ has a negative bottom-left and positive top-right off-diagonal. This gives the results:

Result 12. *If the true model is simultaneous quantity setting, sequential quantity setting can be falsified with either type of instrument.*

If the true model is sequential quantity setting, simultaneous quantity setting, and sequential quantity setting with the wrong first-mover, can both be falsified with either type of instrument.